Time-Optimal Construction of String Synchronizing Sets

Jonas Ellert and Tomasz Kociumaka

Some gems of stringology...

- exact pattern matching...

$$T = \overset{1}{\boxed{\texttt{s t a c k s a t s t a c s a t t r a c k a}}}\overset{n}{} \qquad P = \overset{1}{\boxed{\texttt{a c k}}}\overset{m}{}$$

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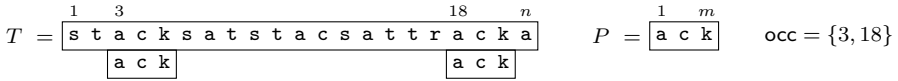

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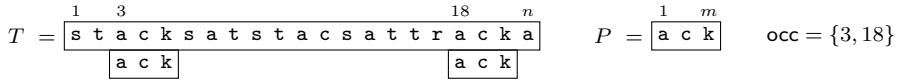
- exact pattern matching... $\mathcal{O}(n)$ time [Knuth et al. '77]



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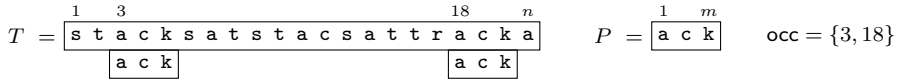

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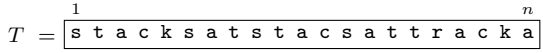

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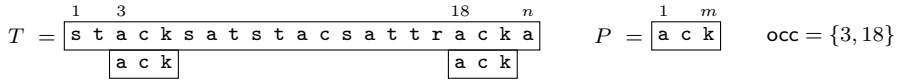

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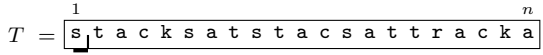

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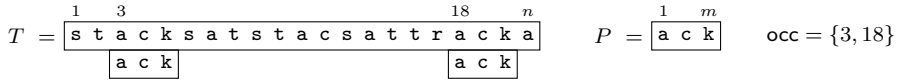

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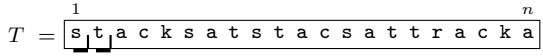

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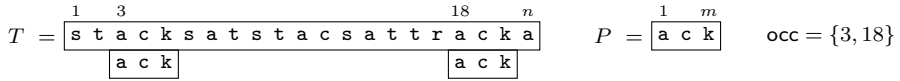

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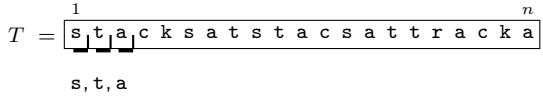

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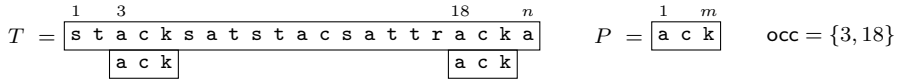

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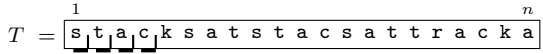

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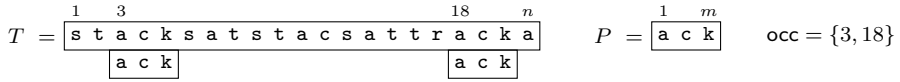

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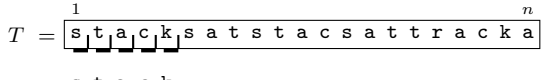

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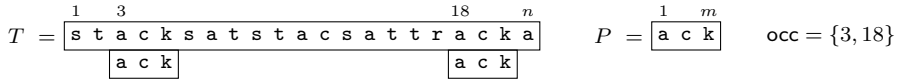

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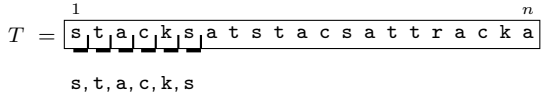

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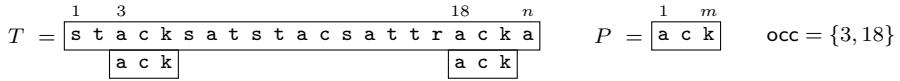

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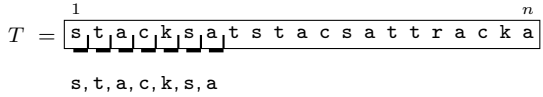

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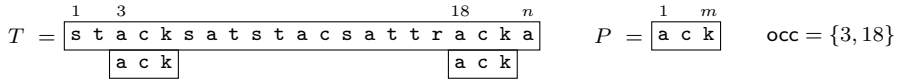

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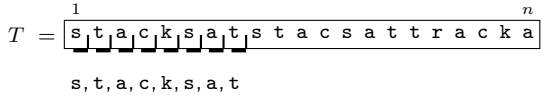

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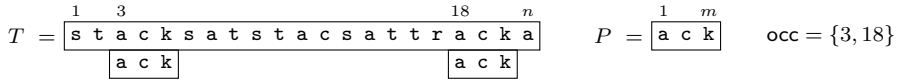

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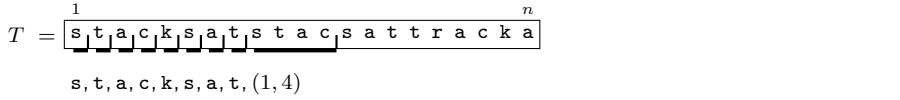

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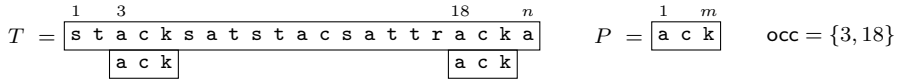

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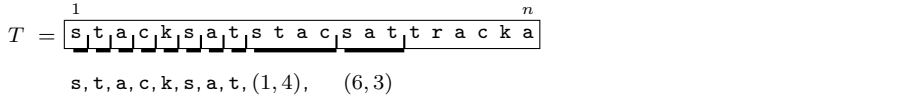

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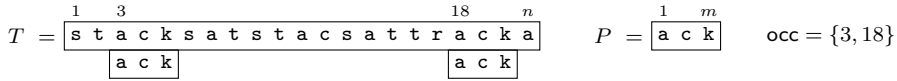

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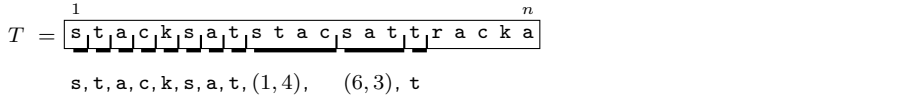

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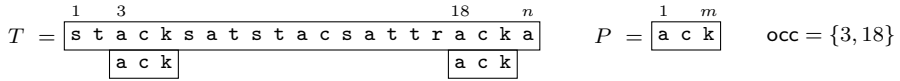

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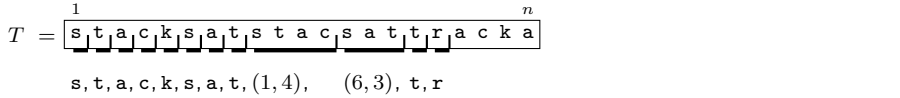

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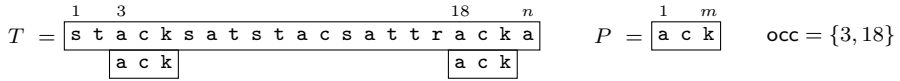

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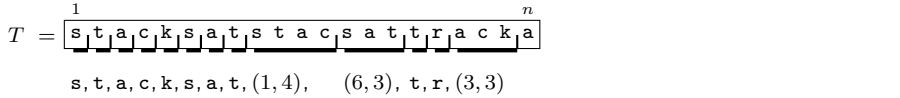

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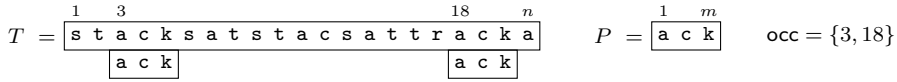

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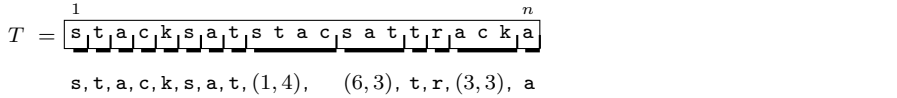

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$$T = \texttt{s t a c k s a t s t a c s a t t r a c k a} \qquad P = \texttt{a c k} \qquad \text{occ} = \{3, 18\}$$

(positions marked above T: 1, 3, 18, n; above P: 1, m; occurrences of $\texttt{a c k}$ shown below T at positions 3 and 18)

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(positions marked above T: 1, n)

$$\texttt{s}, \texttt{t}, \texttt{a}, \texttt{c}, \texttt{k}, \texttt{s}, \texttt{a}, \texttt{t}, (1, 4), \quad (6, 3), \ \texttt{t}, \texttt{r}, (3, 3), \ \texttt{a}$$

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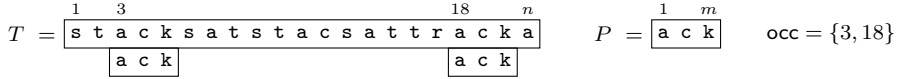

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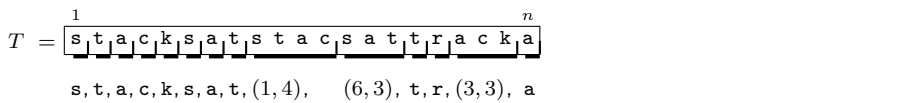

- Longest Common Extension (LCE) data structure...
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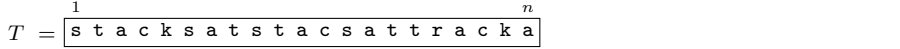

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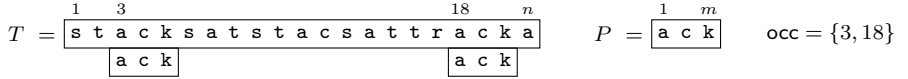

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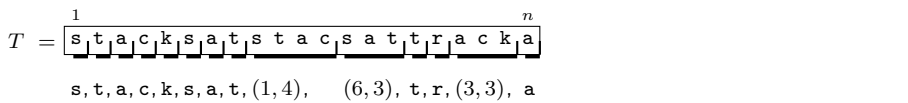

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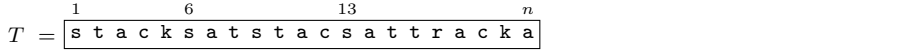

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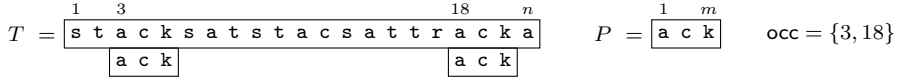

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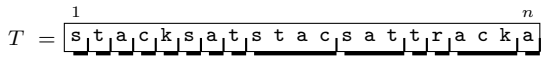

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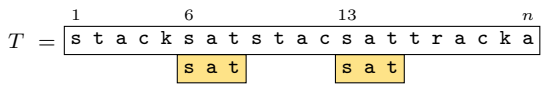

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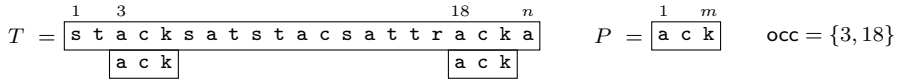

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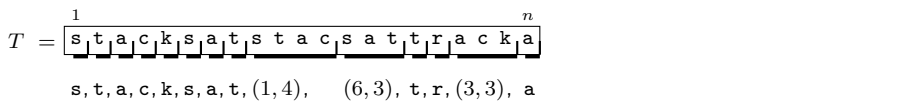

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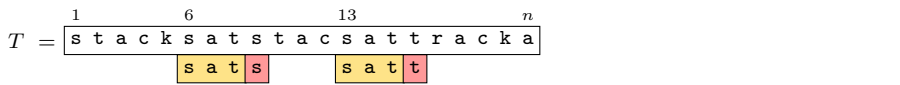

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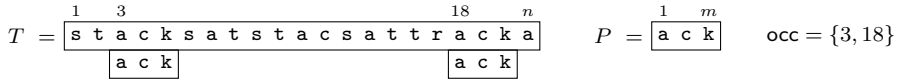

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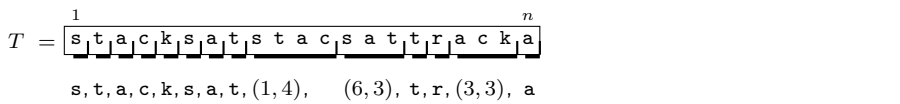

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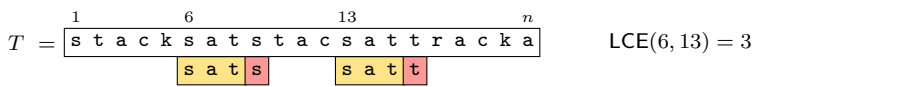

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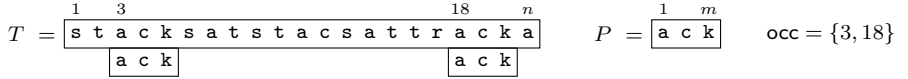

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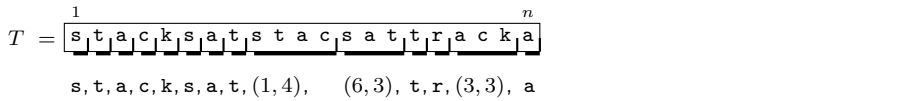

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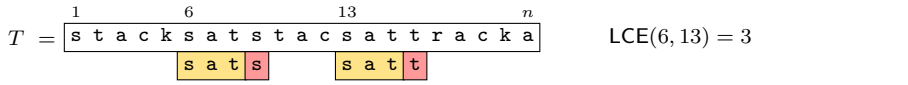

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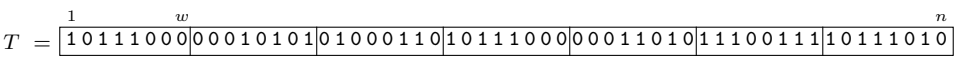

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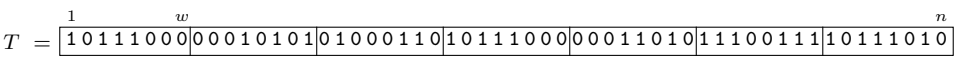

... only $\approx n/w = \Theta(n/\log n)$ words. Why not process in $\mathcal{O}(n/\log n)$ time?

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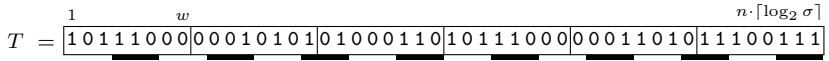

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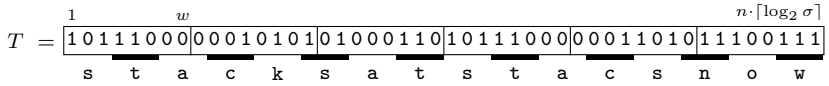

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a	000
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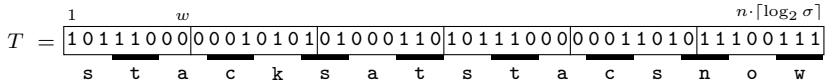

... $\approx n \cdot \lceil \log_2 \sigma \rceil / w$ words. **Truly linear time:** $\mathcal{O}(n \log \sigma / \log n) = \mathcal{O}(n / \log_\sigma n)$

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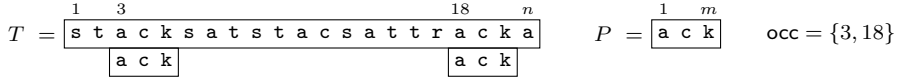

$T =$ s t a c k s a t s t a c s a t t r a c k a $\quad P =$ a c k $\quad \text{occ} = \{3, 18\}$

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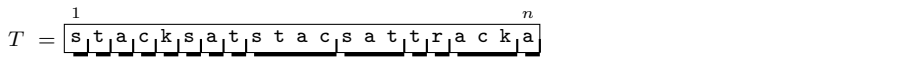

$T =$

s, t, a, c, k, s, a, t, $(1, 4)$, $\quad (6, 3)$, t, r, $(3, 3)$, a

- Longest Common Extension (LCE) data structure... $\mathcal{O}(n)$ construction time [Farach '97]
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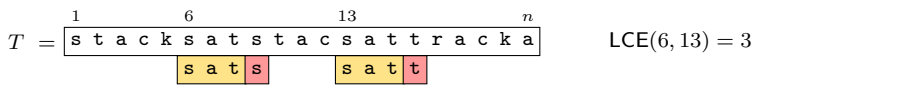

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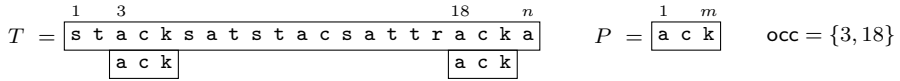

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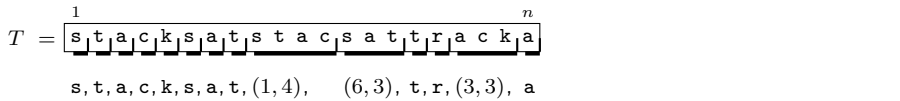

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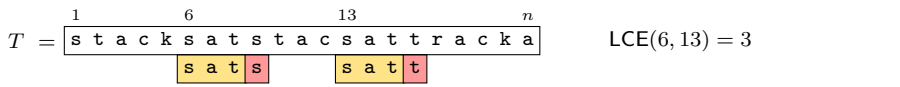

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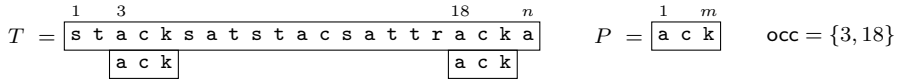

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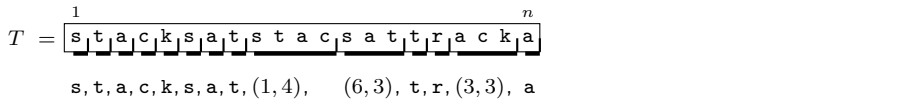

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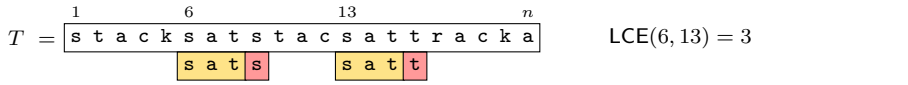

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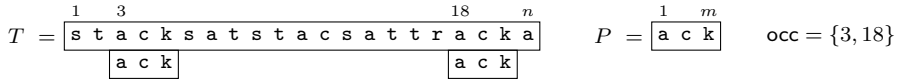

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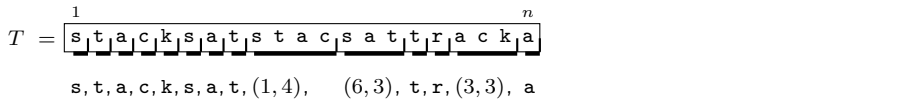

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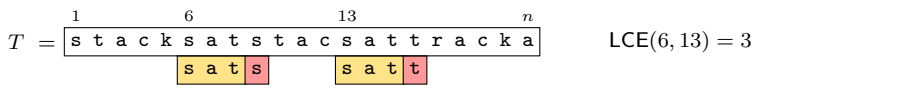

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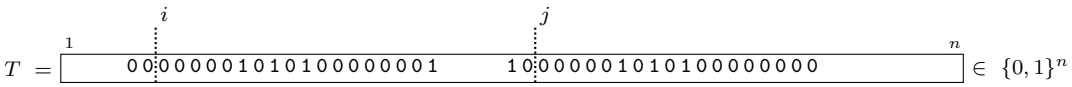

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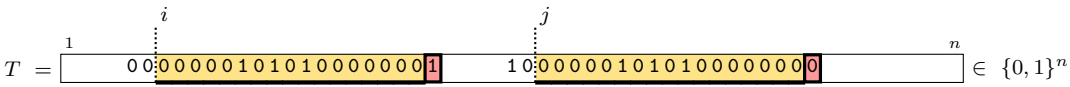

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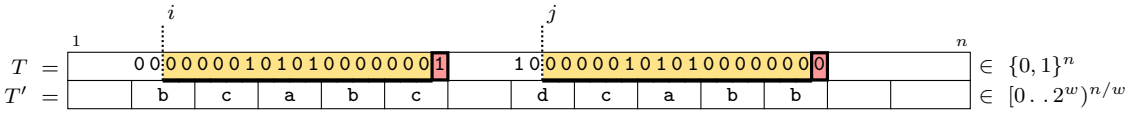

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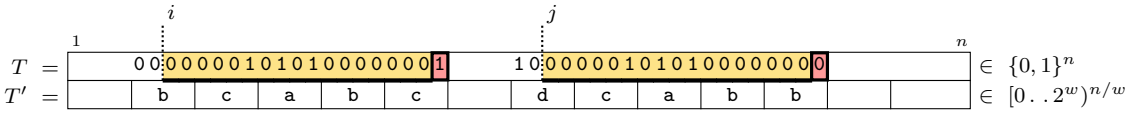

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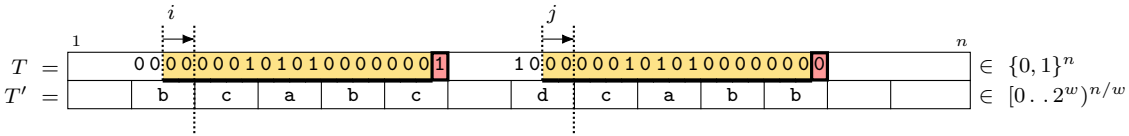

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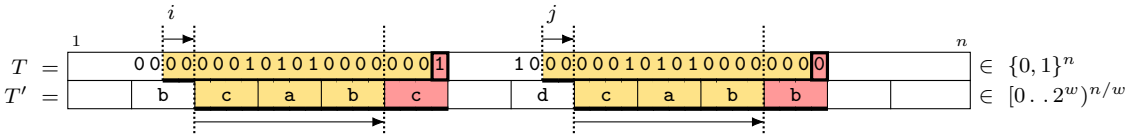

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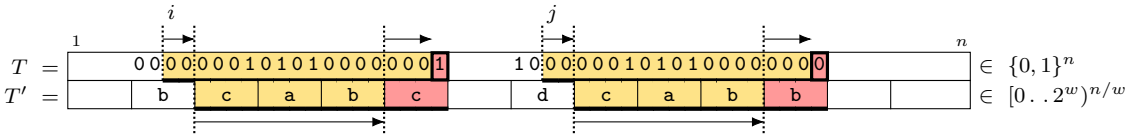

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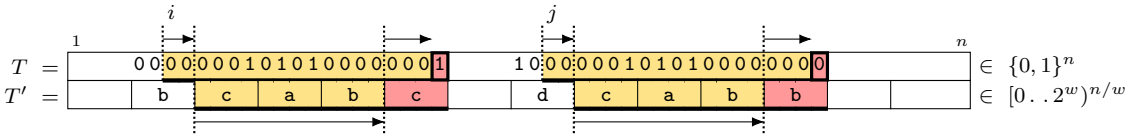

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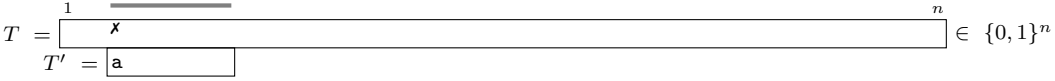

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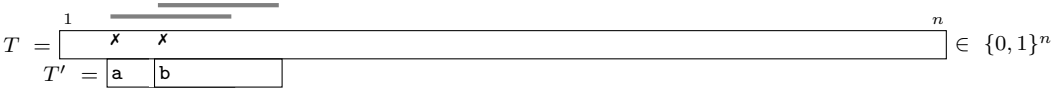

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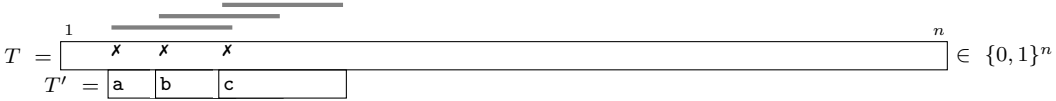

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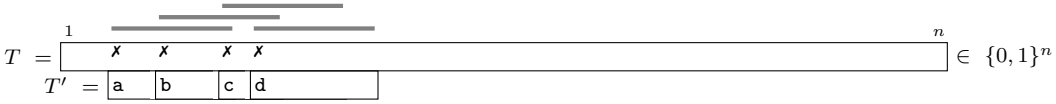

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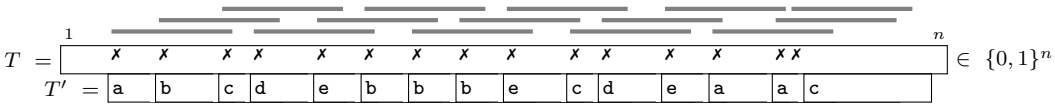

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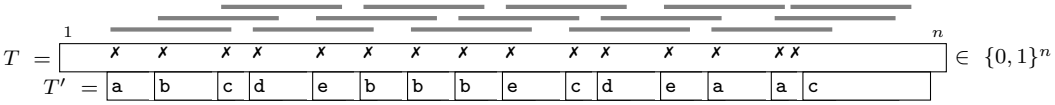

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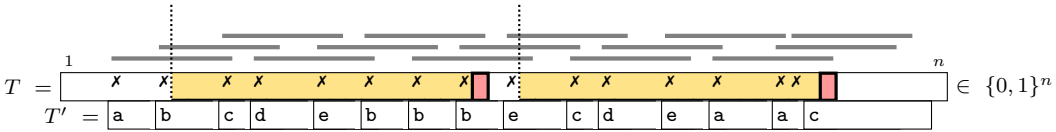

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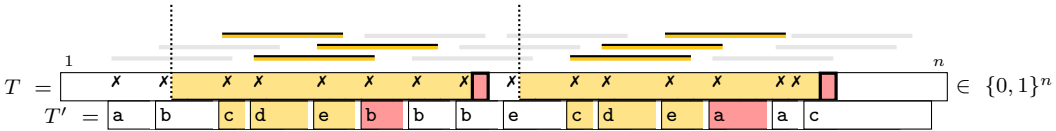

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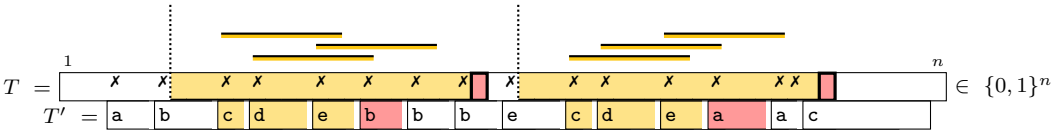

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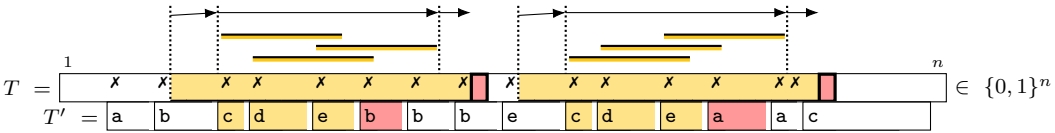

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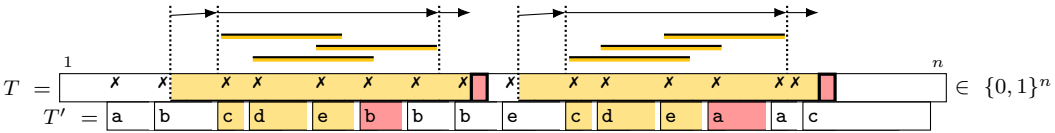

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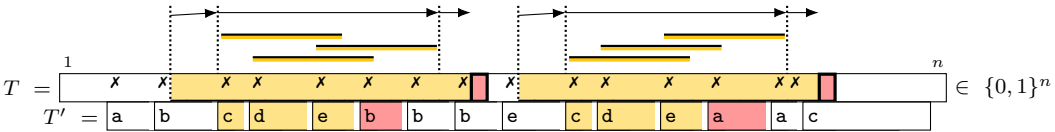

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- some extra conditions for periodic fragments apply

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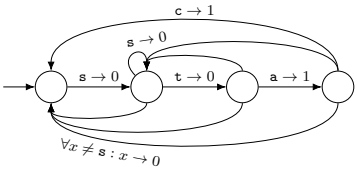
- LCE data structure of T' is of size $\mathcal{O}(n/\tau)$ words

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$



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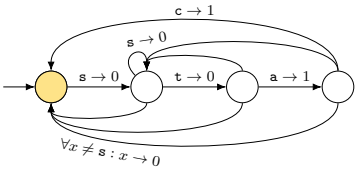

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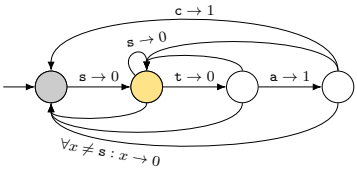

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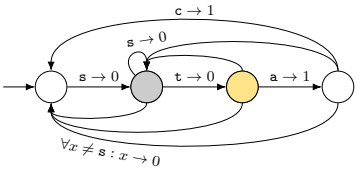

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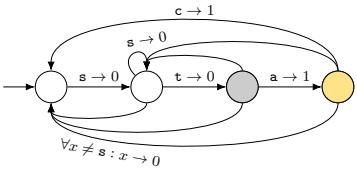

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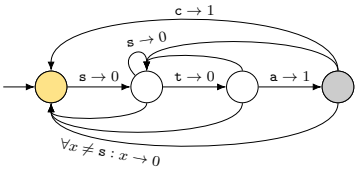

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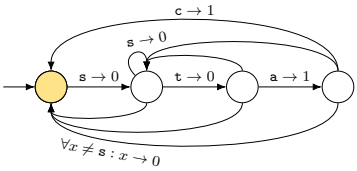

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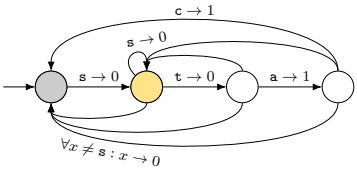

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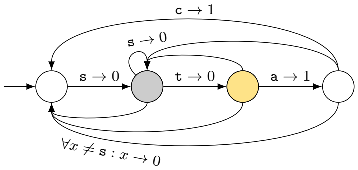

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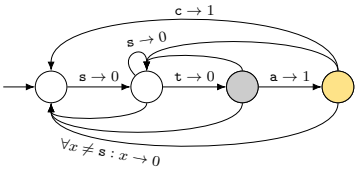

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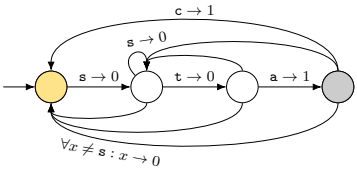

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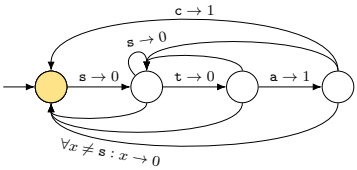

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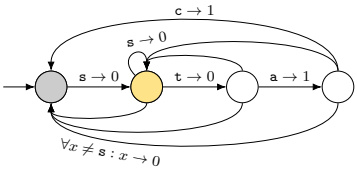

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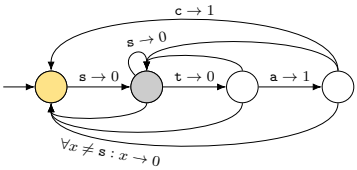
$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0$$



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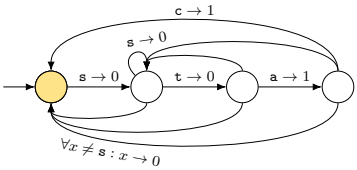

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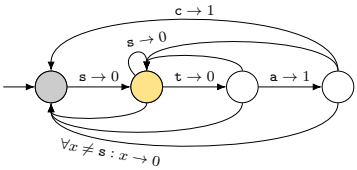

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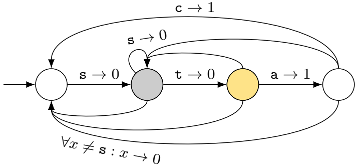
T = s t a r t s t a c k s a t s **t** a c s

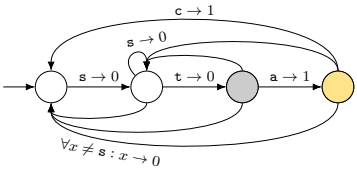
0 0 0 0 0 0 0 0 **1** 0 0 0 0 0 0



Some new tools we developed

$$T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{r} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{k} & \mathtt{s} & \mathtt{a} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{s} \\ \hline \end{array}$$

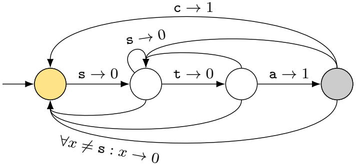
$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$



Some new tools we developed

$T =$ `s t a r t s t a c k s a t s t a c s`

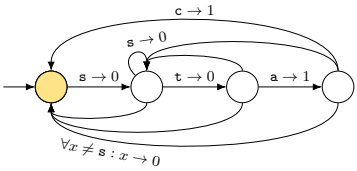
0 0 0 0 0 0 0 0 **1** 0 0 0 0 0 0 0 **1**



Some new tools we developed

$$T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{r} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{k} & \mathtt{s} & \mathtt{a} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{s} \\ \hline \end{array}$$

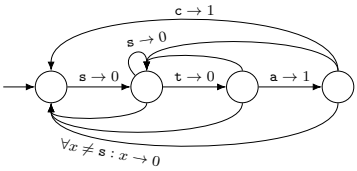
$$\begin{array}{cccccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \end{array}$$



Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0$$

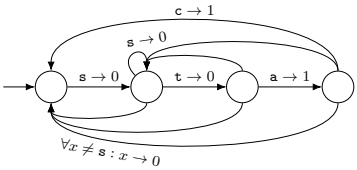


- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0$$

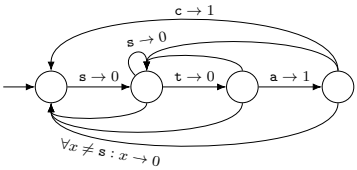


- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time

Some new tools we developed

$$T = \begin{array}{|cccccccccccccccc|}\hline \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{r} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{k} & \mathtt{s} & \mathtt{a} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{s} \\ \hline \end{array}$$

$$\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \end{array}$$

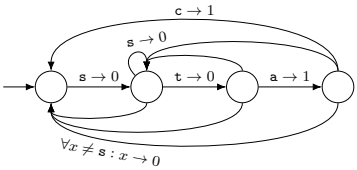


- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time
- also for multiple input/output streams

Some new tools we developed

$$T = \begin{array}{|cccccccccccccccc|}\hline \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{r} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{k} & \mathtt{s} & \mathtt{a} & \mathtt{t} & \mathtt{s} & \mathtt{t} & \mathtt{a} & \mathtt{c} & \mathtt{s} \\ \hline \end{array}$$

$$\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \end{array}$$

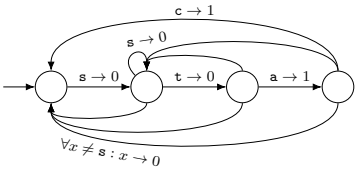


- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time
- also for multiple input/output streams

Custom sparse encoding:

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$
$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0$$



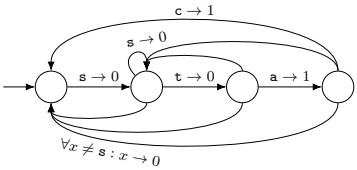
- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time
- also for multiple input/output streams

Custom sparse encoding:

$$T = \boxed{\texttt{0 0 0 0 2 0 0 6 0 0 0 1 0 0 0 0 0 9 0 0 4 4}}$$

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$
$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{1}\ 0$$



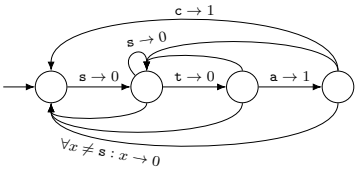
- preprocess q-state transducer in $\mathcal{O}(qn^{\varepsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time
- also for multiple input/output streams

Custom sparse encoding:

$$T = \boxed{\texttt{0 0 0 0 2 0 0 6 0 0 0 1 0 0 0 0 0 9 0 0 4 4}}$$
$$\underbrace{}_{4}\ \underbrace{}_{2}\ \underbrace{}_{3}\ \underbrace{}_{5}\ \underbrace{}_{2}$$

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$
$$\texttt{0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0}$$



- preprocess q-state transducer in $\mathcal{O}(qn^{\epsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time
- also for multiple input/output streams

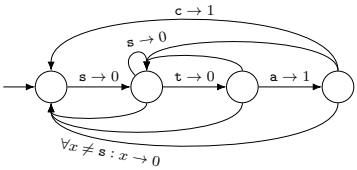
Custom sparse encoding:

$$T = \underbrace{\texttt{0 0 0 0}}_{4}\,\underbrace{\texttt{2 0 0}}_{2}\,\underbrace{\texttt{6 0 0 0}}_{3}\,\underbrace{\texttt{1 0 0 0 0 0}}_{5}\,\underbrace{\texttt{9 0 0}}_{2}\,\texttt{4 4}$$

$$\text{enc}(T) = \texttt{0}\cdot\delta(4)\cdot\texttt{1}\cdot\delta(2)\cdot\texttt{0}\cdot\delta(2)\cdot\texttt{1}\cdot\delta(6)\cdot\texttt{0}\cdot\delta(3)\cdot\texttt{1}\cdot\delta(1)\cdot\texttt{0}\cdot\delta(5)\cdot\texttt{1}\cdot\delta(9)\cdot\texttt{0}\cdot\delta(2)\cdot\texttt{1}\cdot\delta(4)\cdot\texttt{1}\cdot\delta(4)$$

Some new tools we developed

$$T = \boxed{\texttt{s t a r t s t a c k s a t s t a c s}}$$
$$\texttt{0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0}$$



- preprocess q-state transducer in $\mathcal{O}(qn^{\epsilon})$ time
- run transducer in $\mathcal{O}(n/\log_{\sigma} n)$ time or $\mathcal{O}(|\text{enc}(T)|/\log n)$
- also for multiple input/output streams

Custom sparse encoding:

$$T = \boxed{\underbrace{\texttt{0 0 0 0}}_{4}\ \underbrace{\texttt{2 0 0}}_{2}\ \underbrace{\texttt{6 0 0 0}}_{3}\ \underbrace{\texttt{1 0 0 0 0 0}}_{5}\ \underbrace{\texttt{9 0 0}}_{2}\ \texttt{4 4}}$$

$$\text{enc}(T) = \texttt{0}\cdot\delta(4)\cdot\texttt{1}\cdot\delta(2)\cdot\texttt{0}\cdot\delta(2)\cdot\texttt{1}\cdot\delta(6)\cdot\texttt{0}\cdot\delta(3)\cdot\texttt{1}\cdot\delta(1)\cdot\texttt{0}\cdot\delta(5)\cdot\texttt{1}\cdot\delta(9)\cdot\texttt{0}\cdot\delta(2)\cdot\texttt{1}\cdot\delta(4)\cdot\texttt{1}\cdot\delta(4)$$

Conclusion

- string synchronizing sets in $\mathcal{O}(n/\log_\sigma n)$ time for all τ
- truly linear time transducers
- sparse encoding for even faster transducers

Conclusion

- string synchronizing sets in $\mathcal{O}(n/\log_\sigma n)$ time for all τ
- truly linear time transducers
- sparse encoding for even faster transducers

Open questions:

- complexity gap $\mathcal{O}(n/\log_\sigma n)$ vs. $\mathcal{O}(n \log \sigma / \sqrt{\log n})$, see [Kempa+Kociumaka '25]
- faster algorithms for $w \gg \log n$, can we get $\mathcal{O}(n \log \sigma / w)$ time?

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