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Structural Parameterization of Steiner Tree Packing

Niko Hastrich[†], Kirill Simonov[‡]

ETH Zürich [†]
University of Bergen [‡]

Def. **S**teiner **T**ree **P**acking

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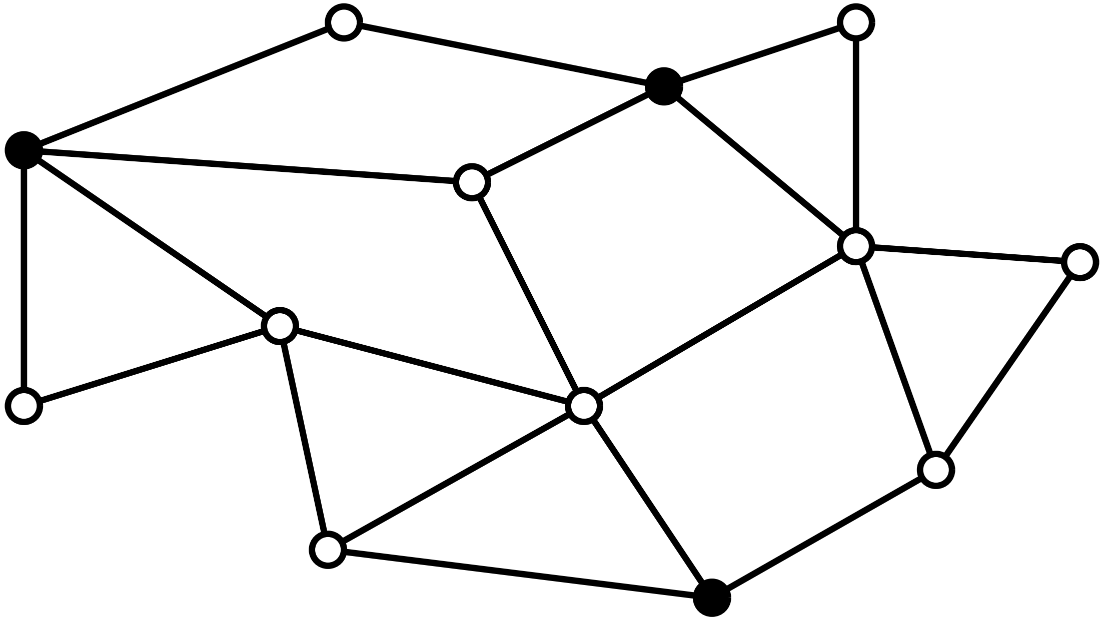

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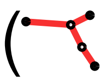
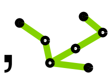
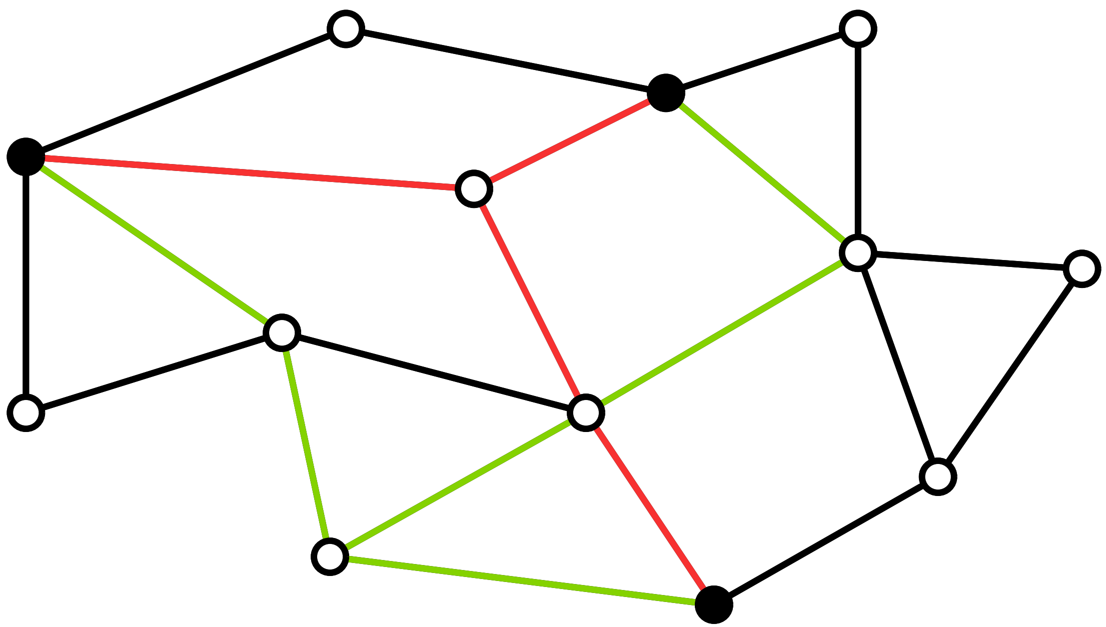
Output: Edge-disjoint, connected subgraphs (, ) connecting T



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Theorem:
STP is FPT by tree-cut width as well as fracture number.

Generalized Steiner Tree Packing
GSTP

Edge-Disjoint Paths
EDP

Steiner Tree Packing
STP

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GSTP

EDGE-DISJOINT PATHS
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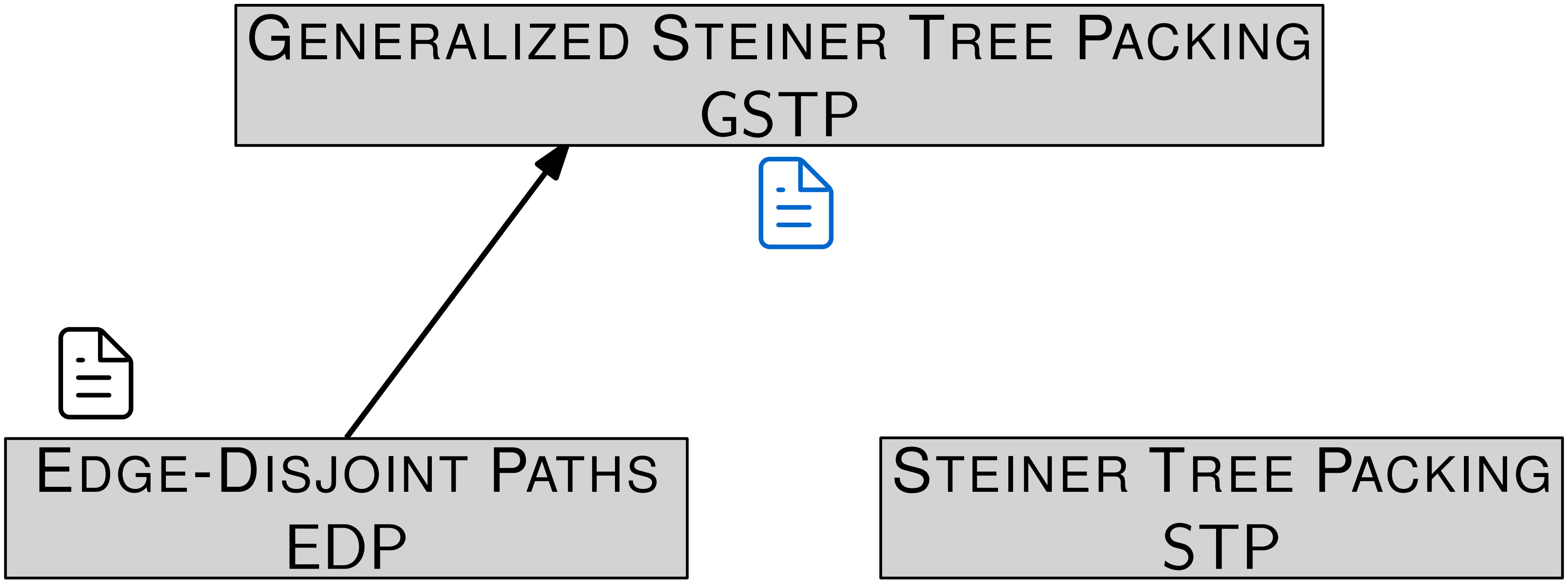
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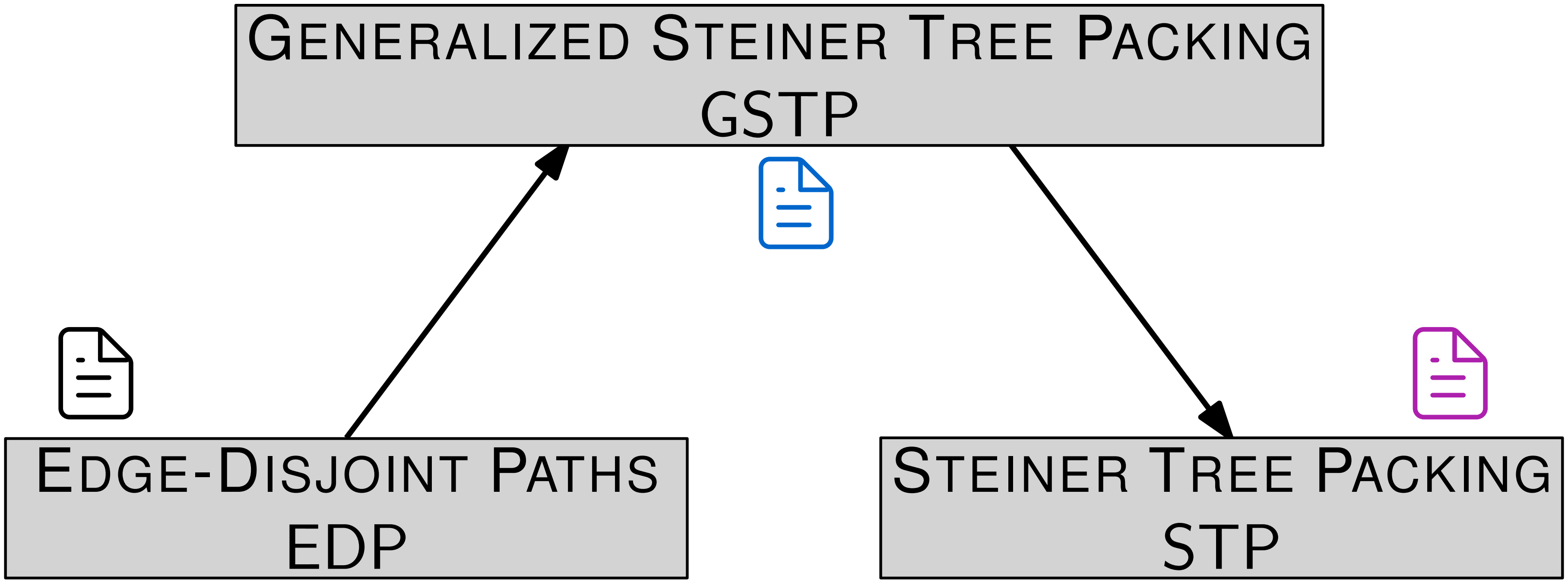

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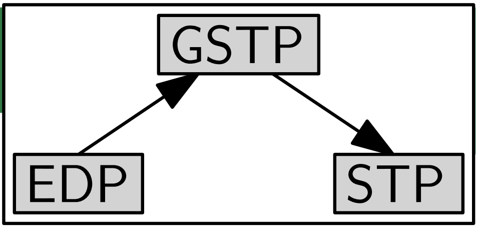

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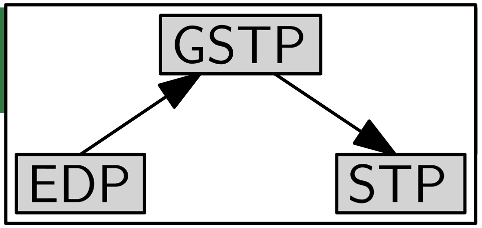
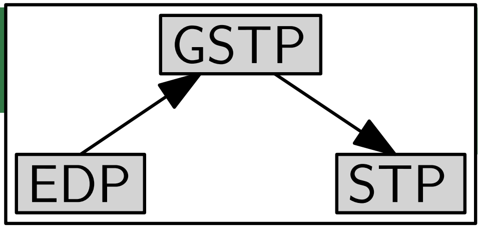
Def. **E**dge **D**isjoint **P**aths





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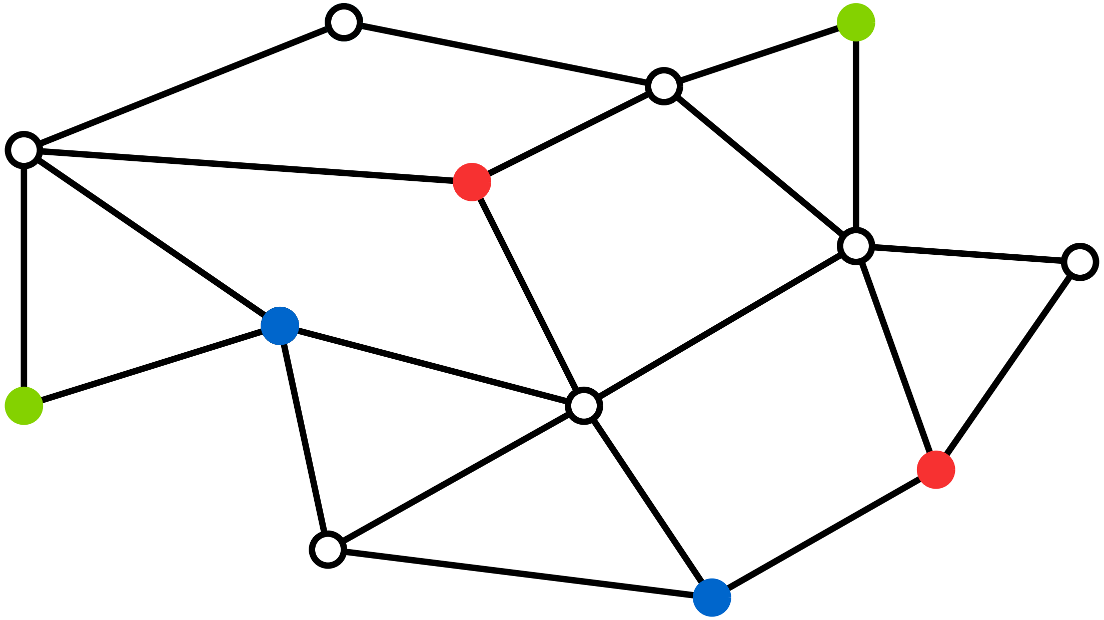

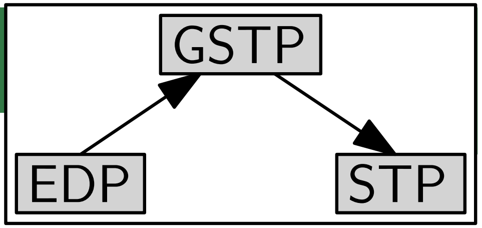

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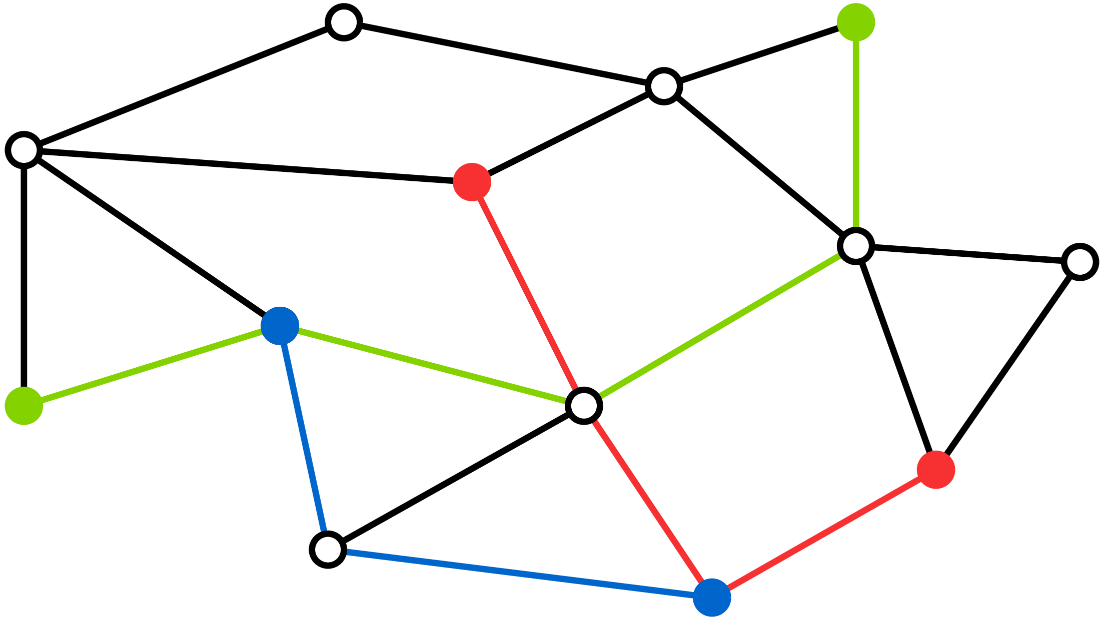

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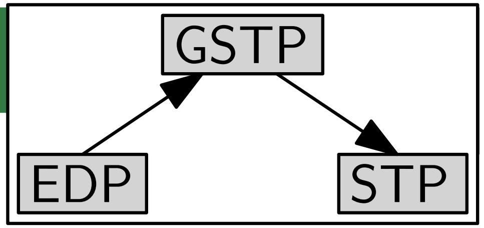

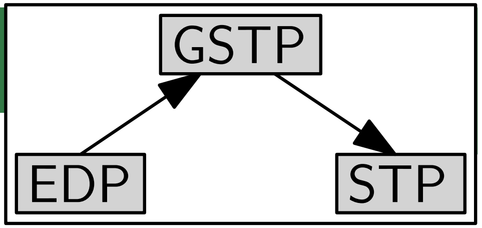

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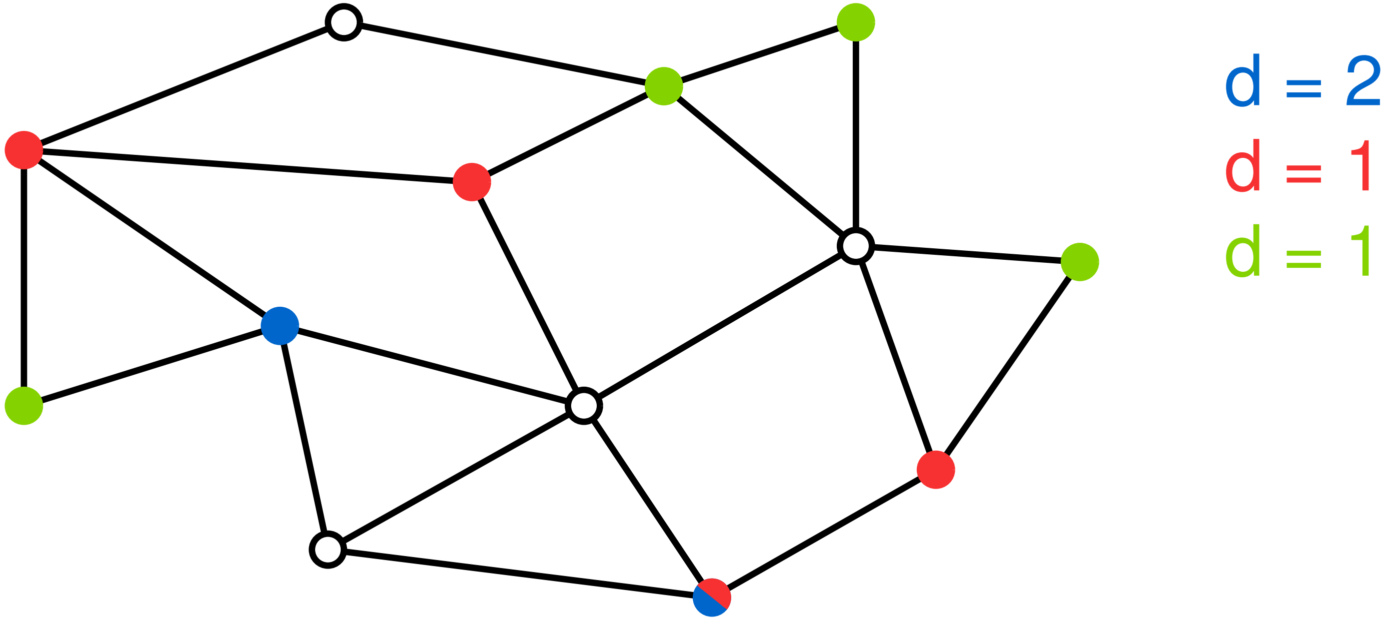

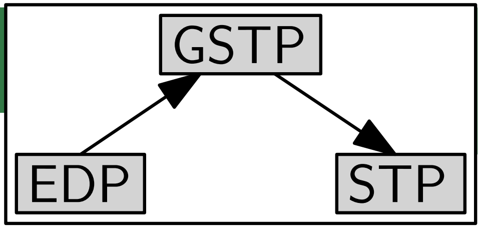

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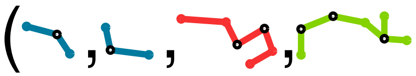
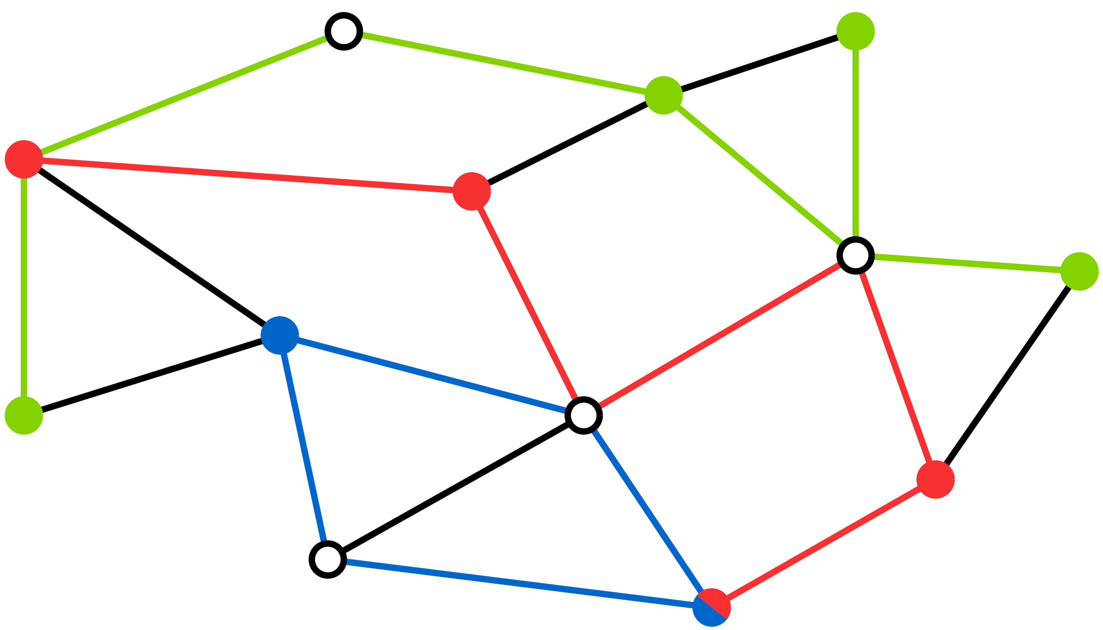
() connecting **their terminal set**



d = 2
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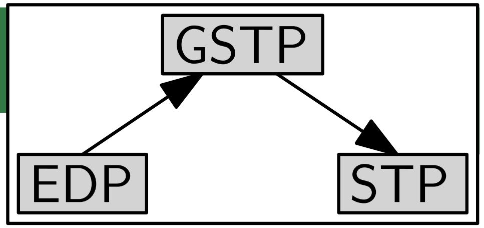

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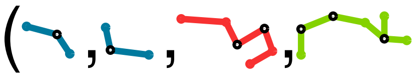
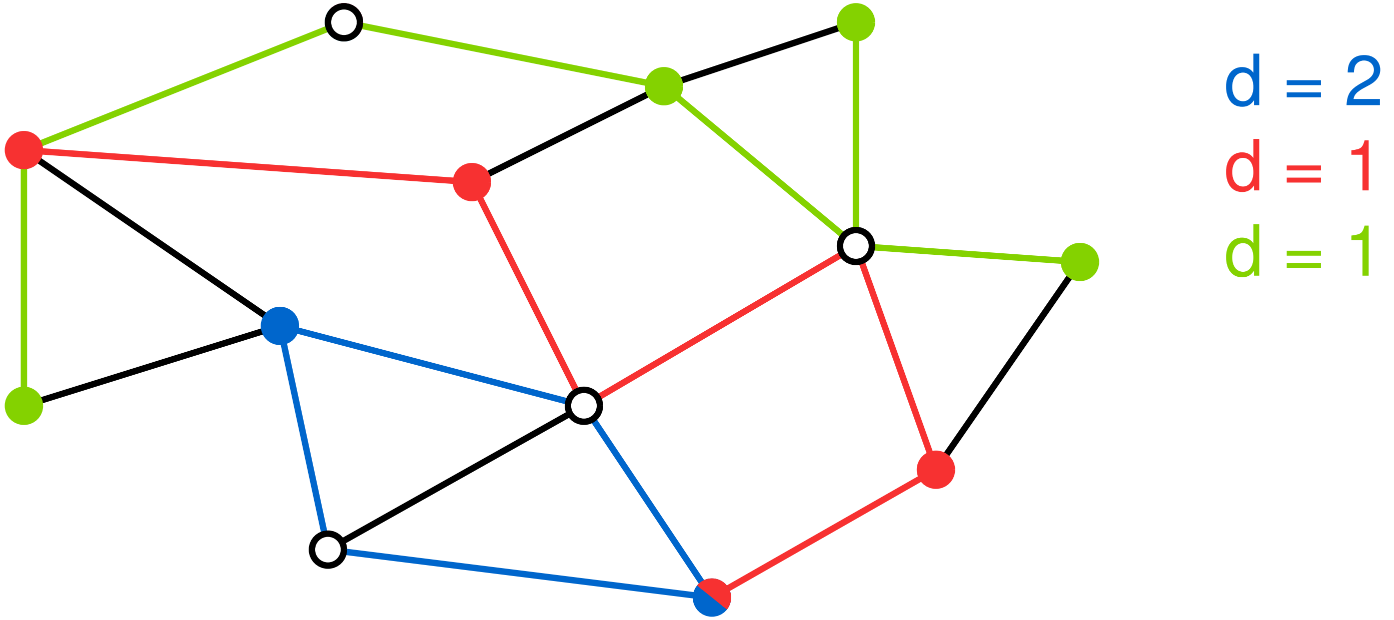
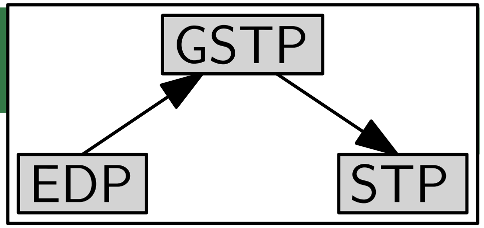
Output: Edge-disjoint, connected subgraphs () connecting **their terminal set**

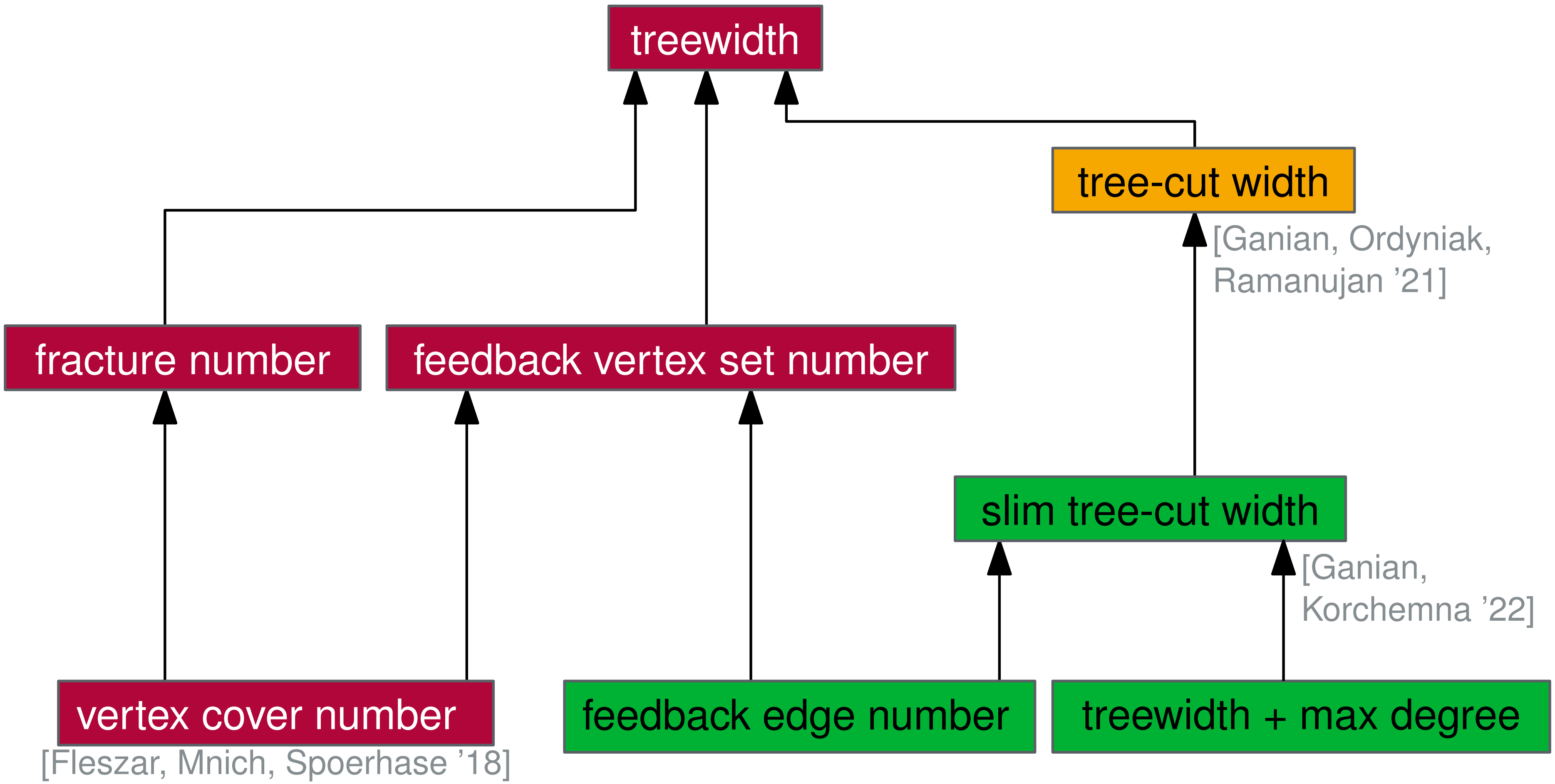
Contains EDP and STP as special cases.



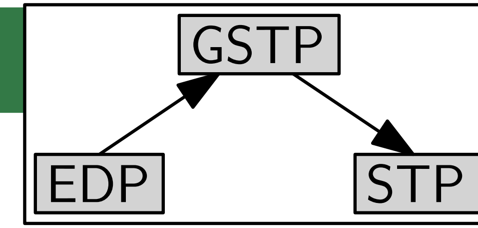


EDP by Input Graph

Known Results:



paraNP-hard W[1]-hard FPT



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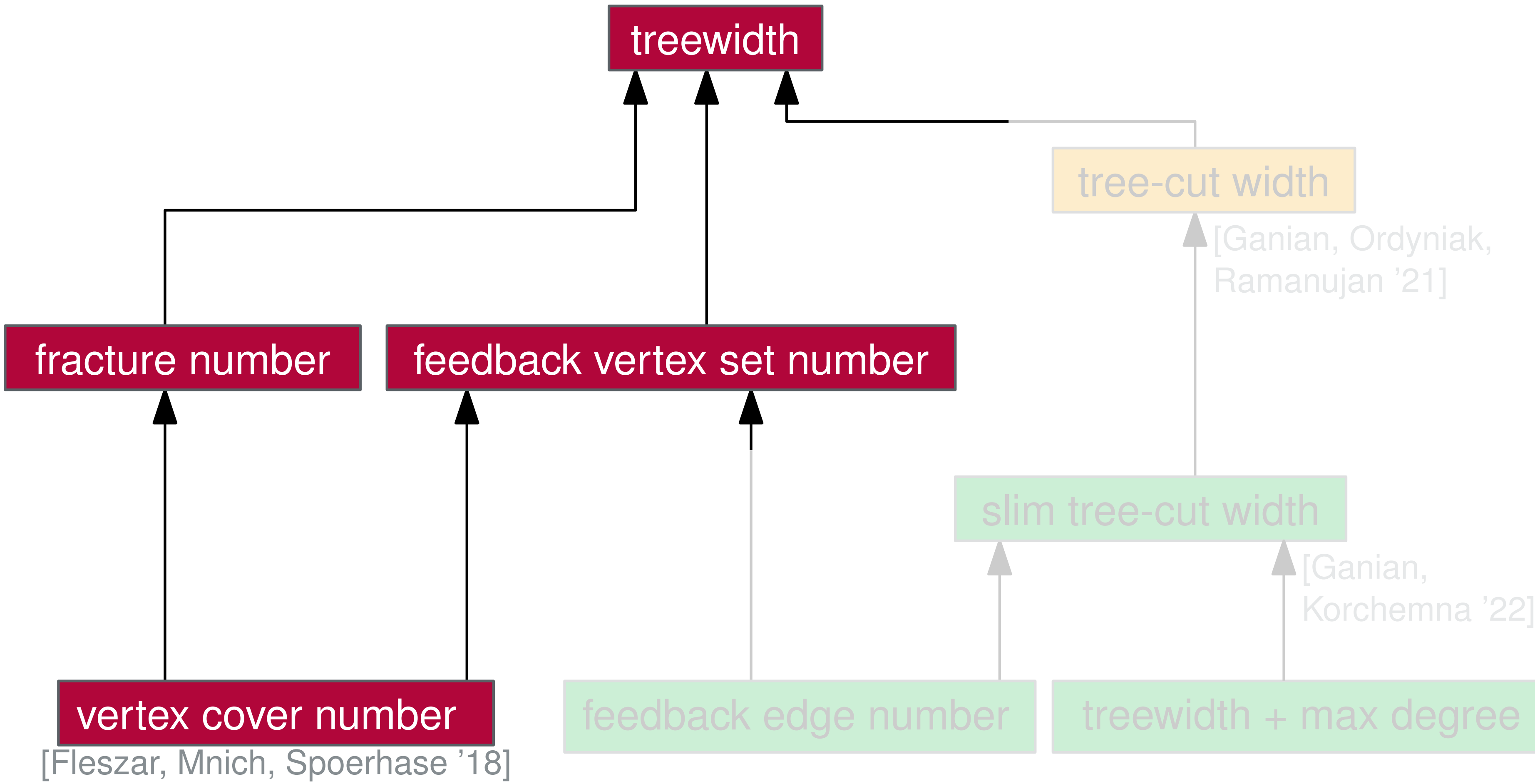

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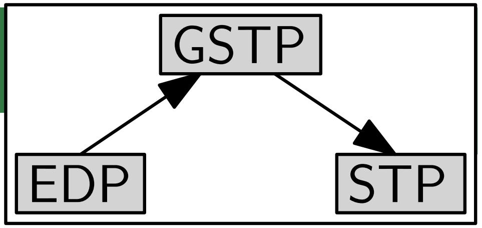

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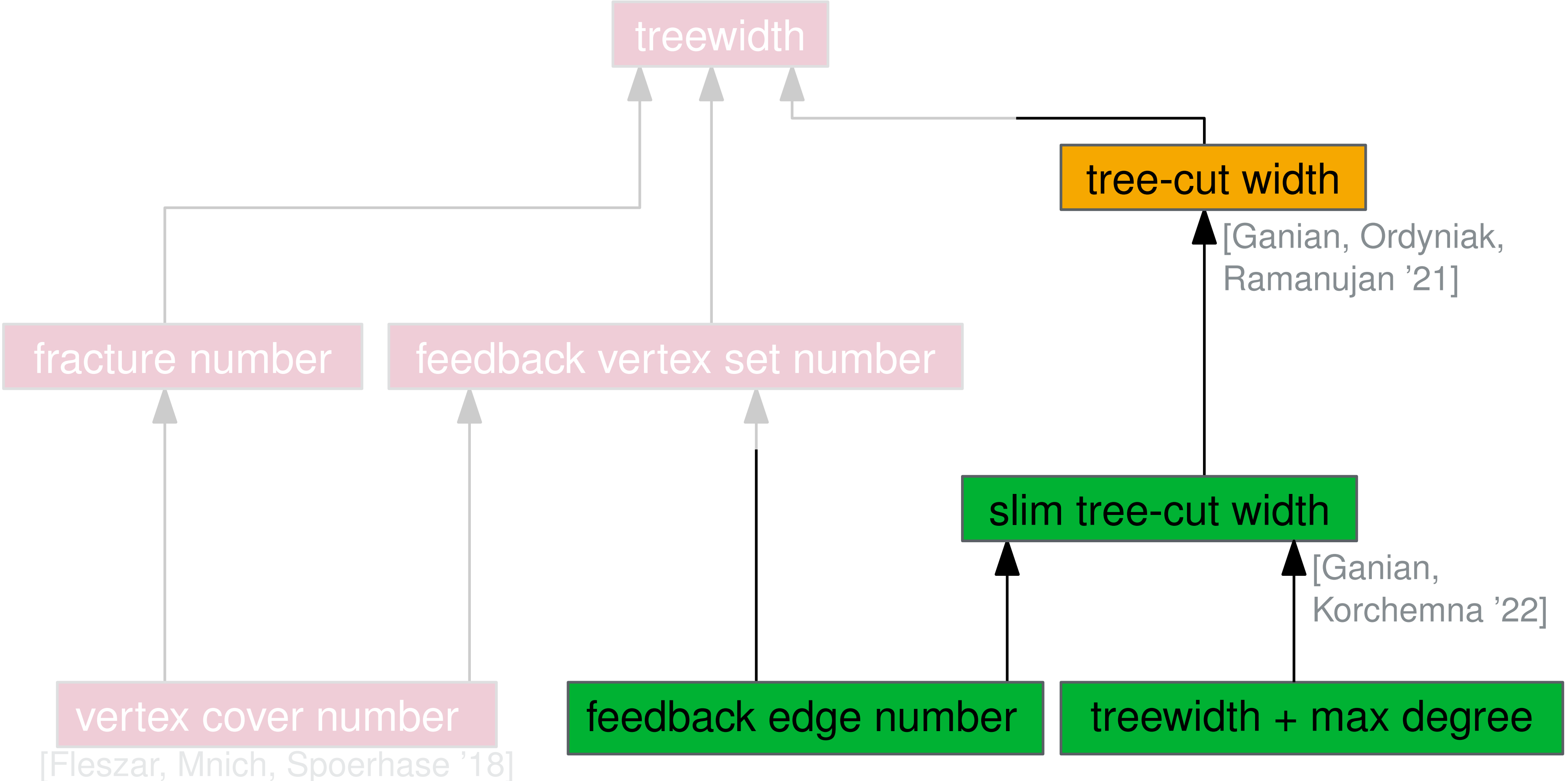

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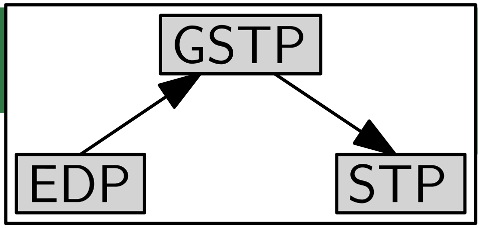

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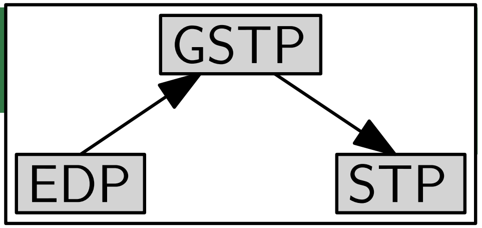

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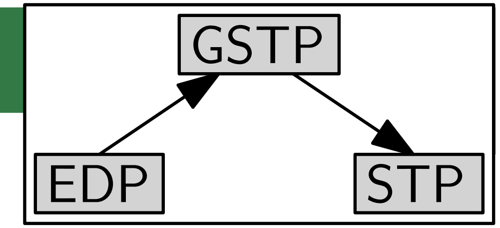
Parameterize by *augmented* graph $G^{\mathcal{T}} := G + \mathcal{T}$.



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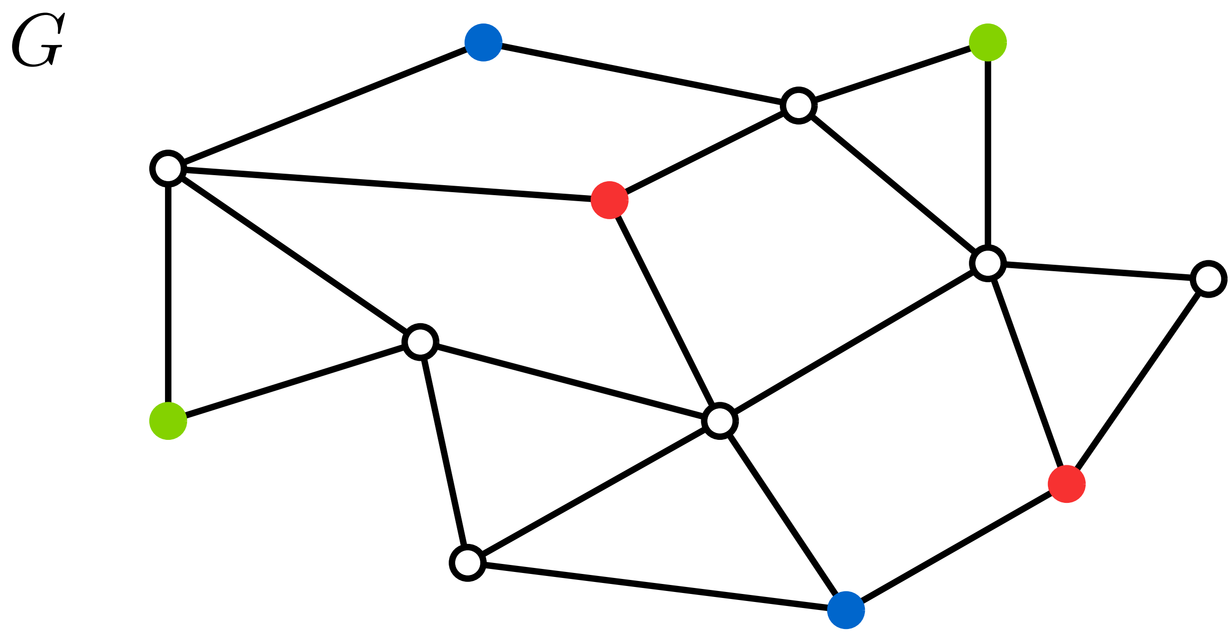

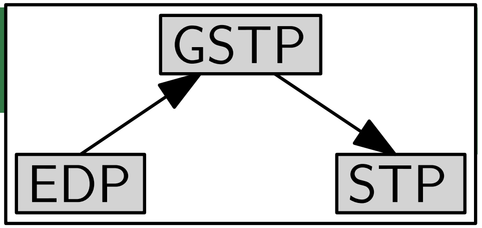

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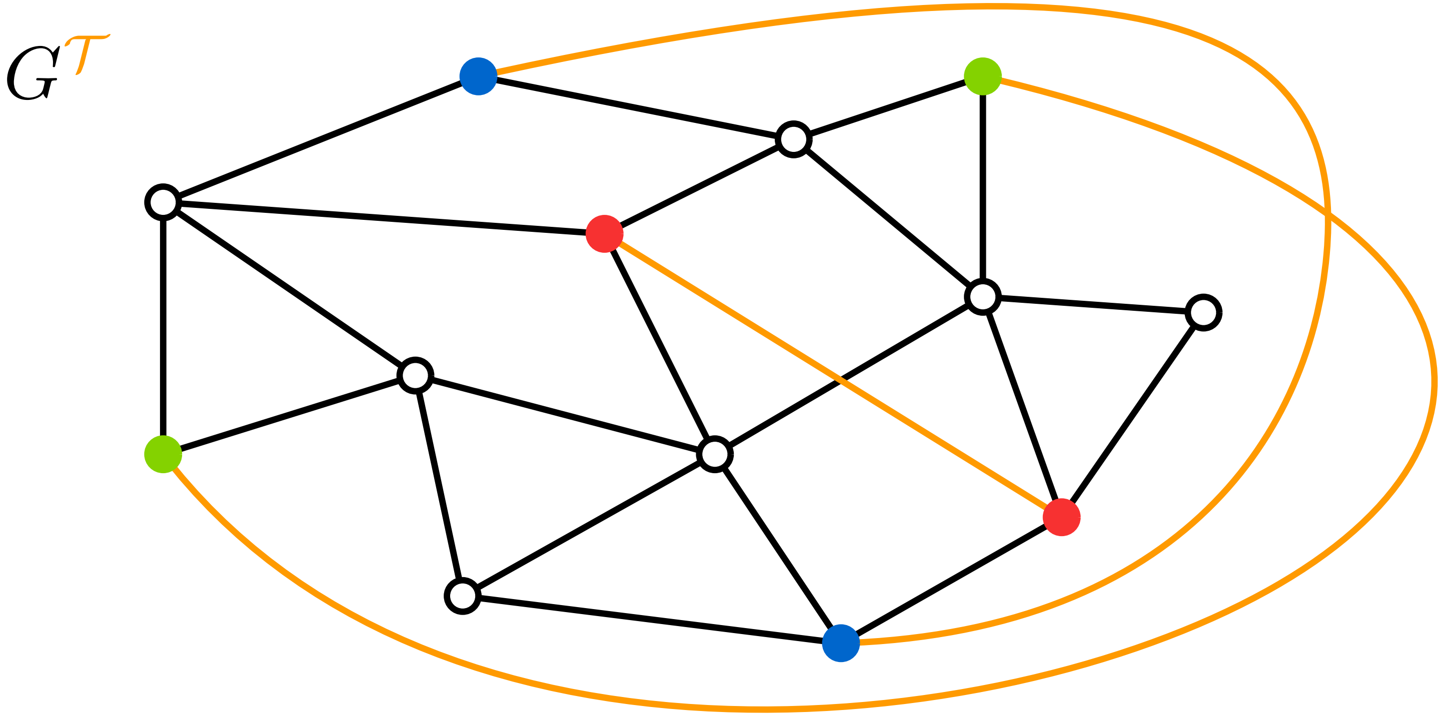

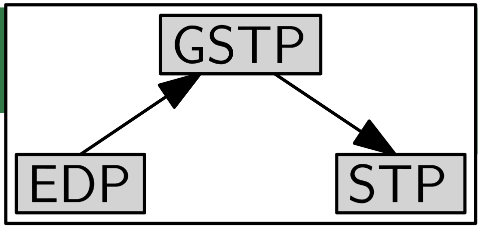

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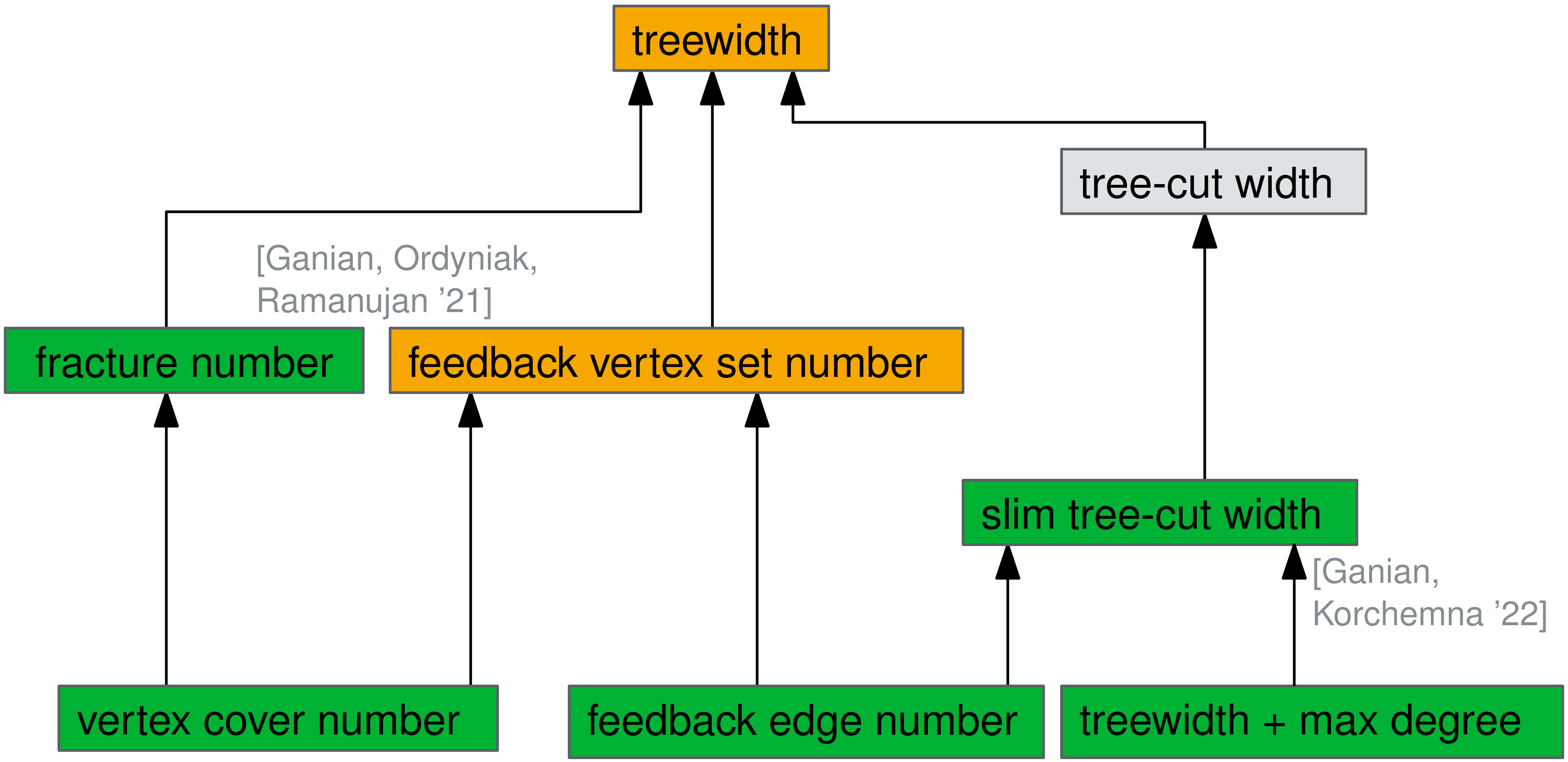

W[1]-hard FPT unknown

Augmentation for GSTP

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Two natural approaches:

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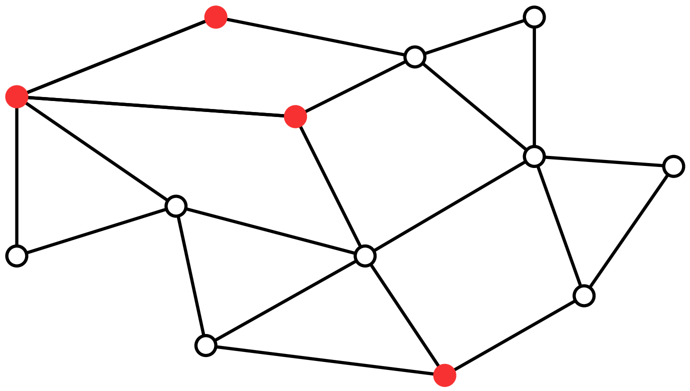
clique-augmented graph



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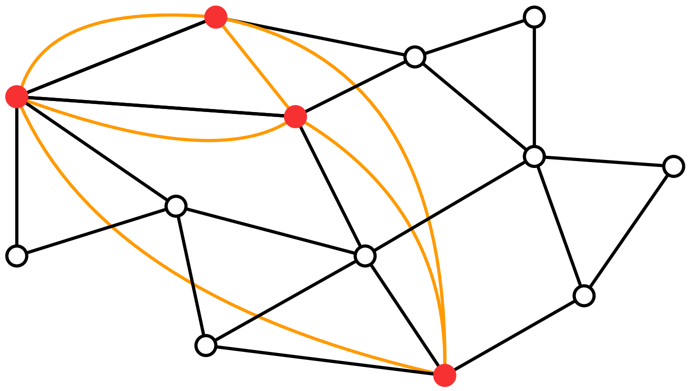

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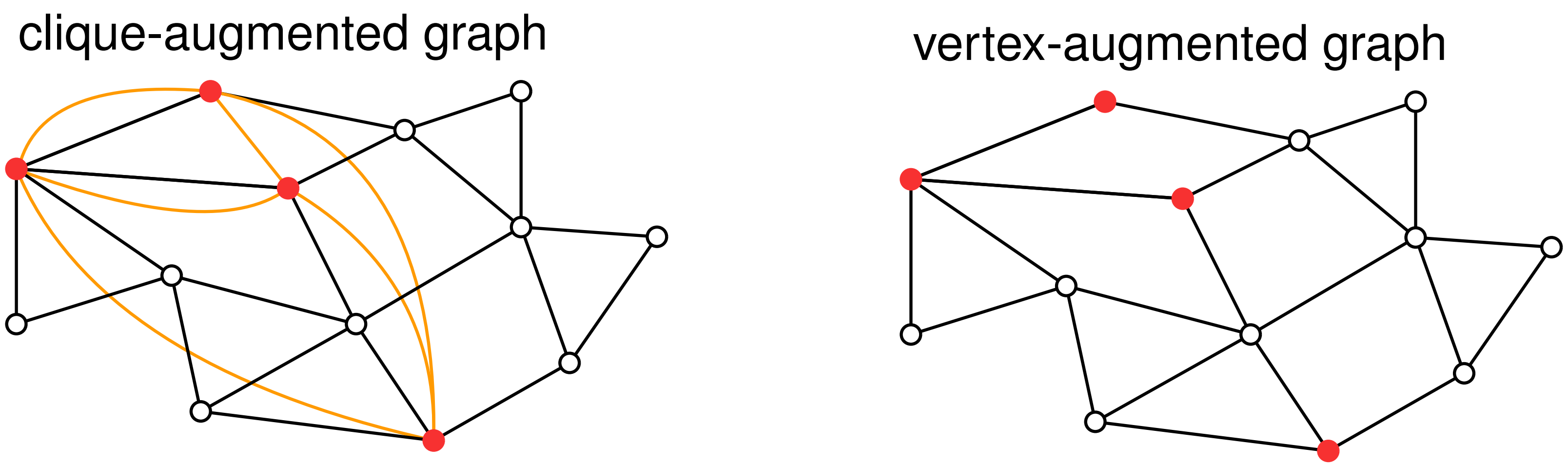

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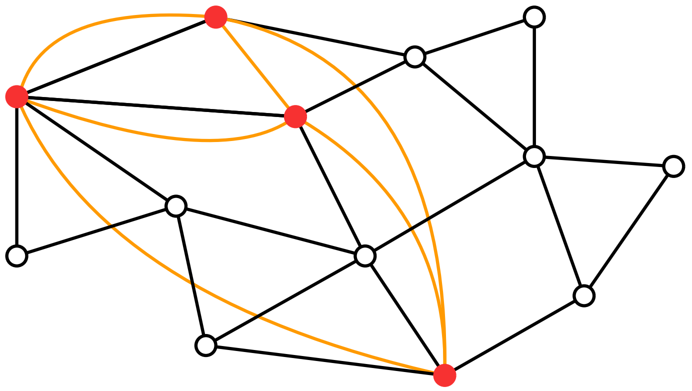
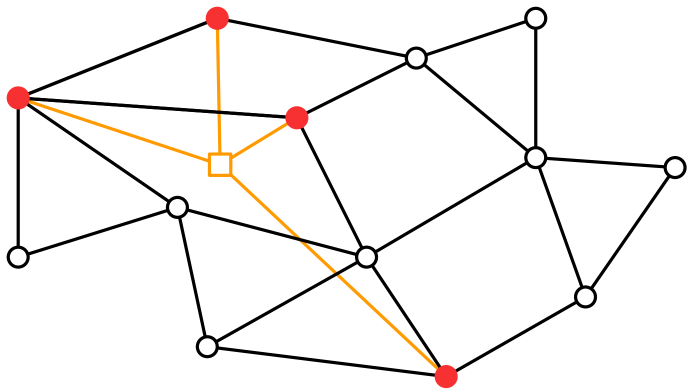


vertex-augmented graph



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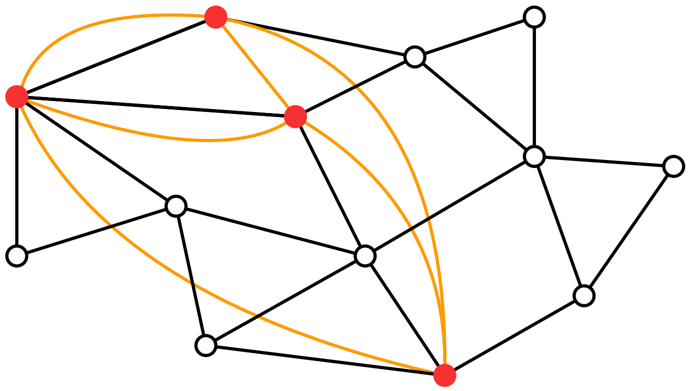

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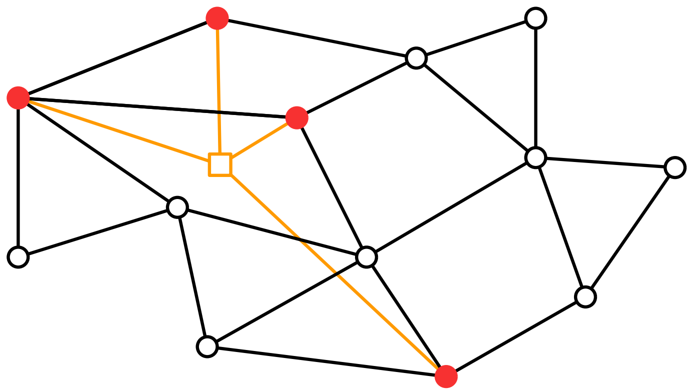

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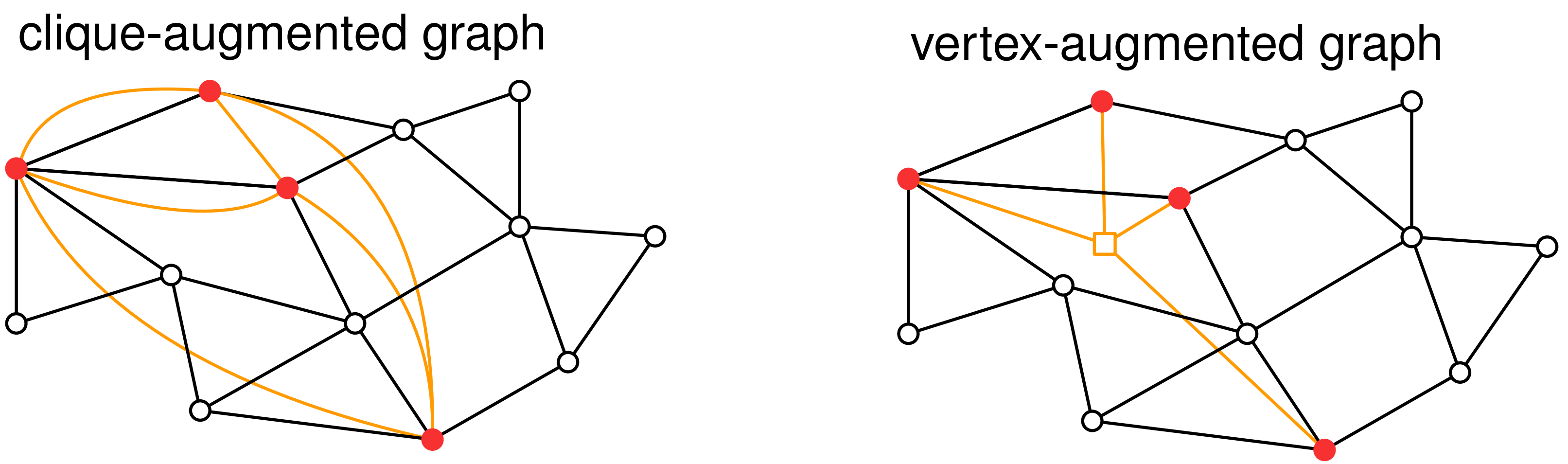

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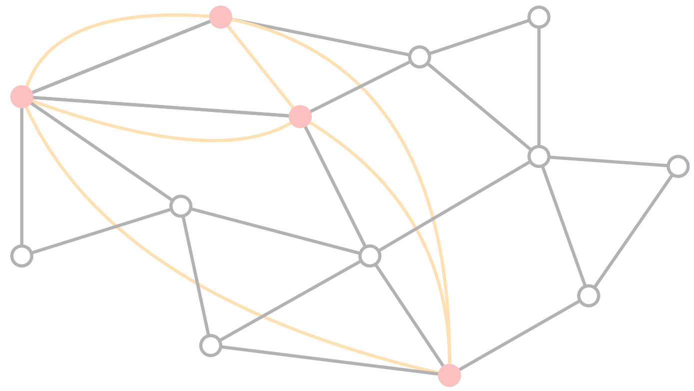
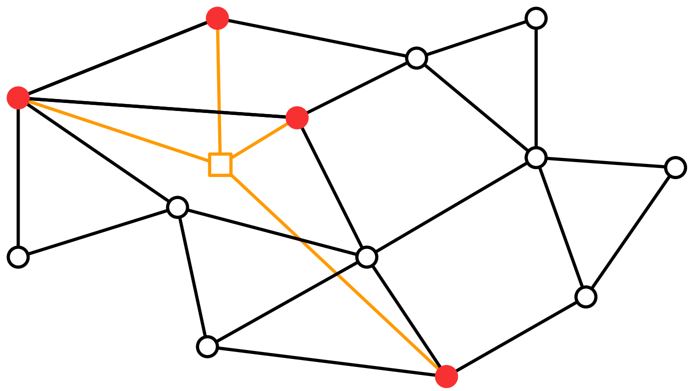


vertex-augmented graph



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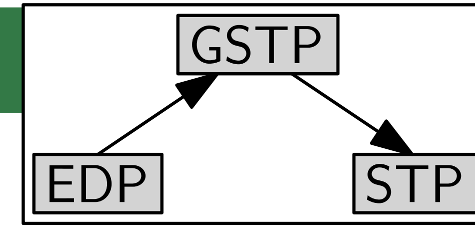
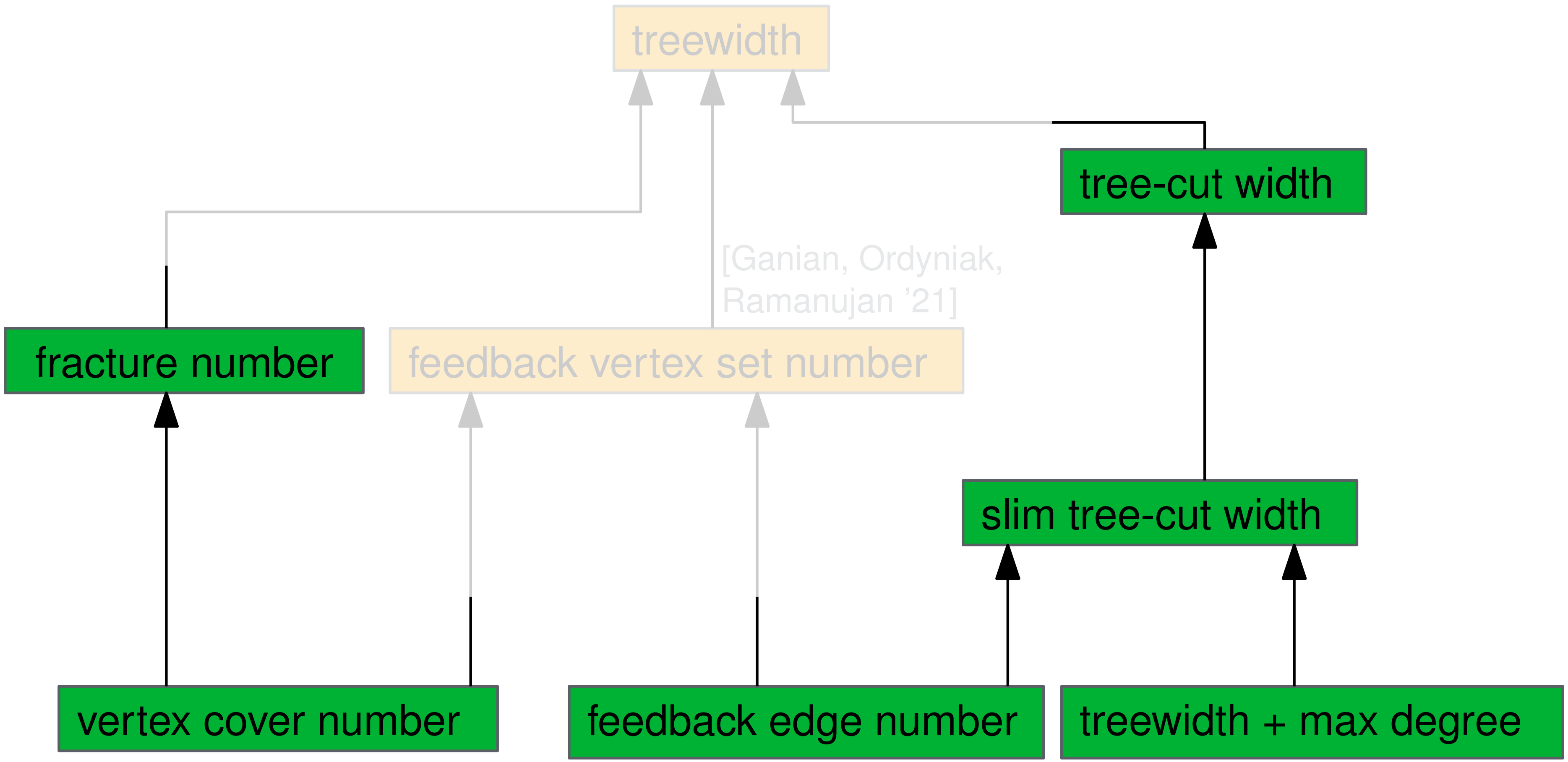
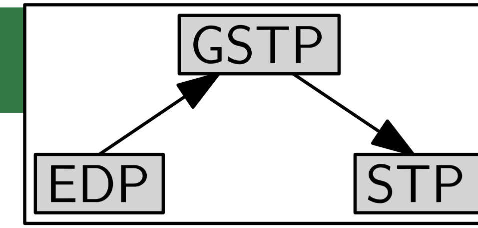


GSTP by Augmented Graph

Our Results:



W[1]-hard FPT



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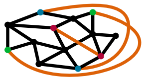

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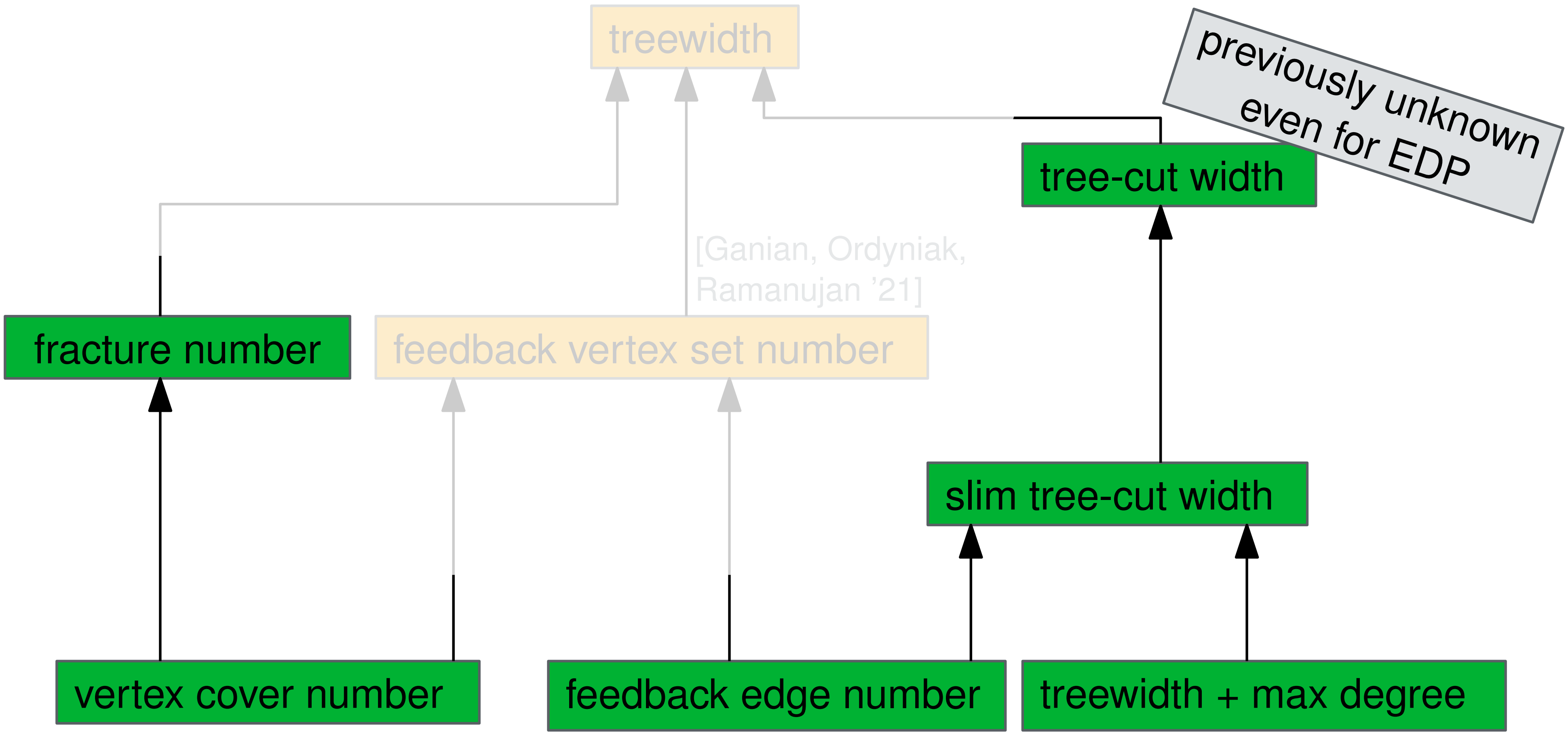

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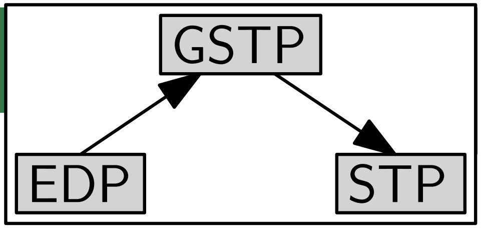

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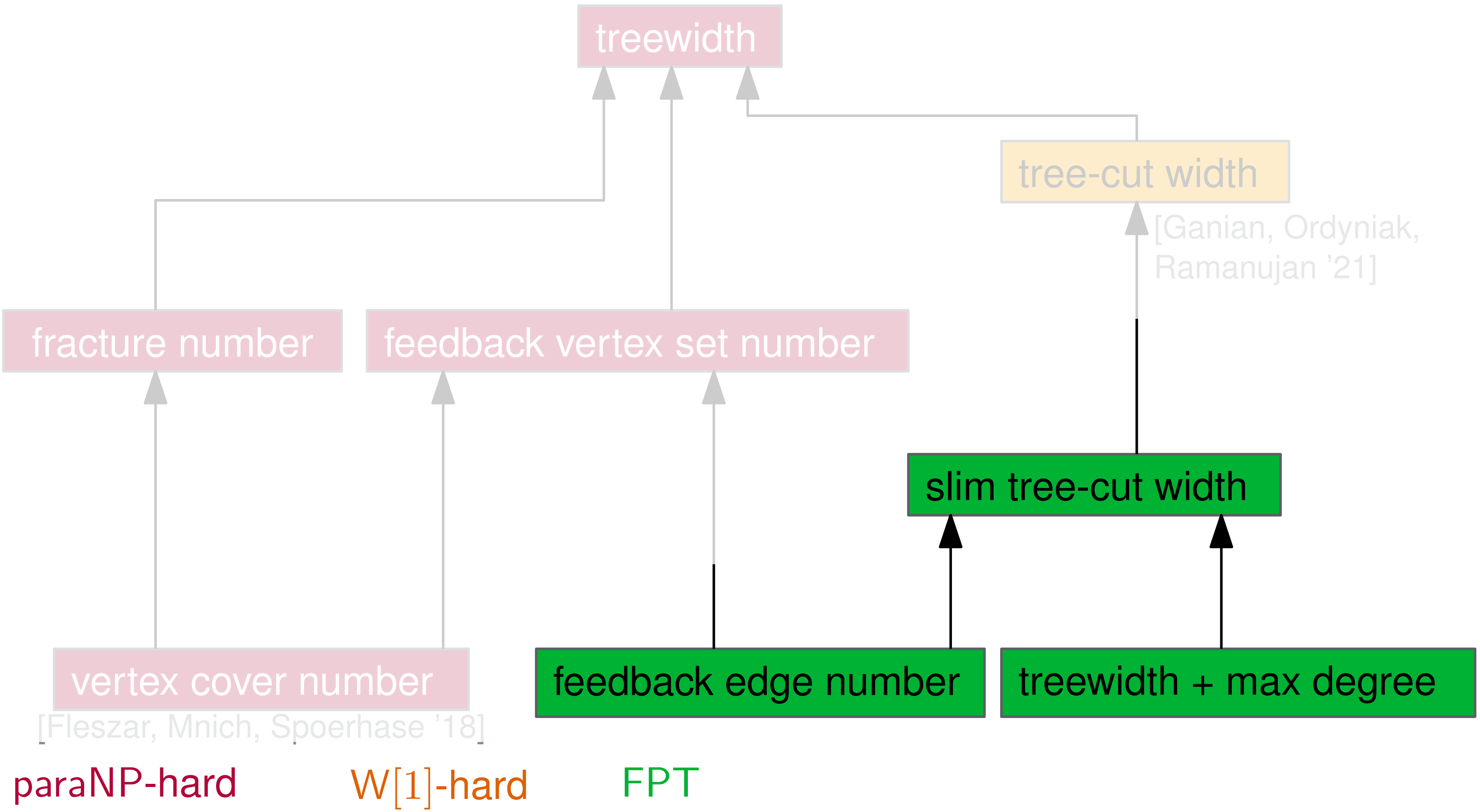

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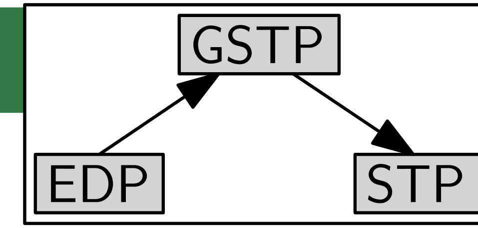
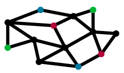
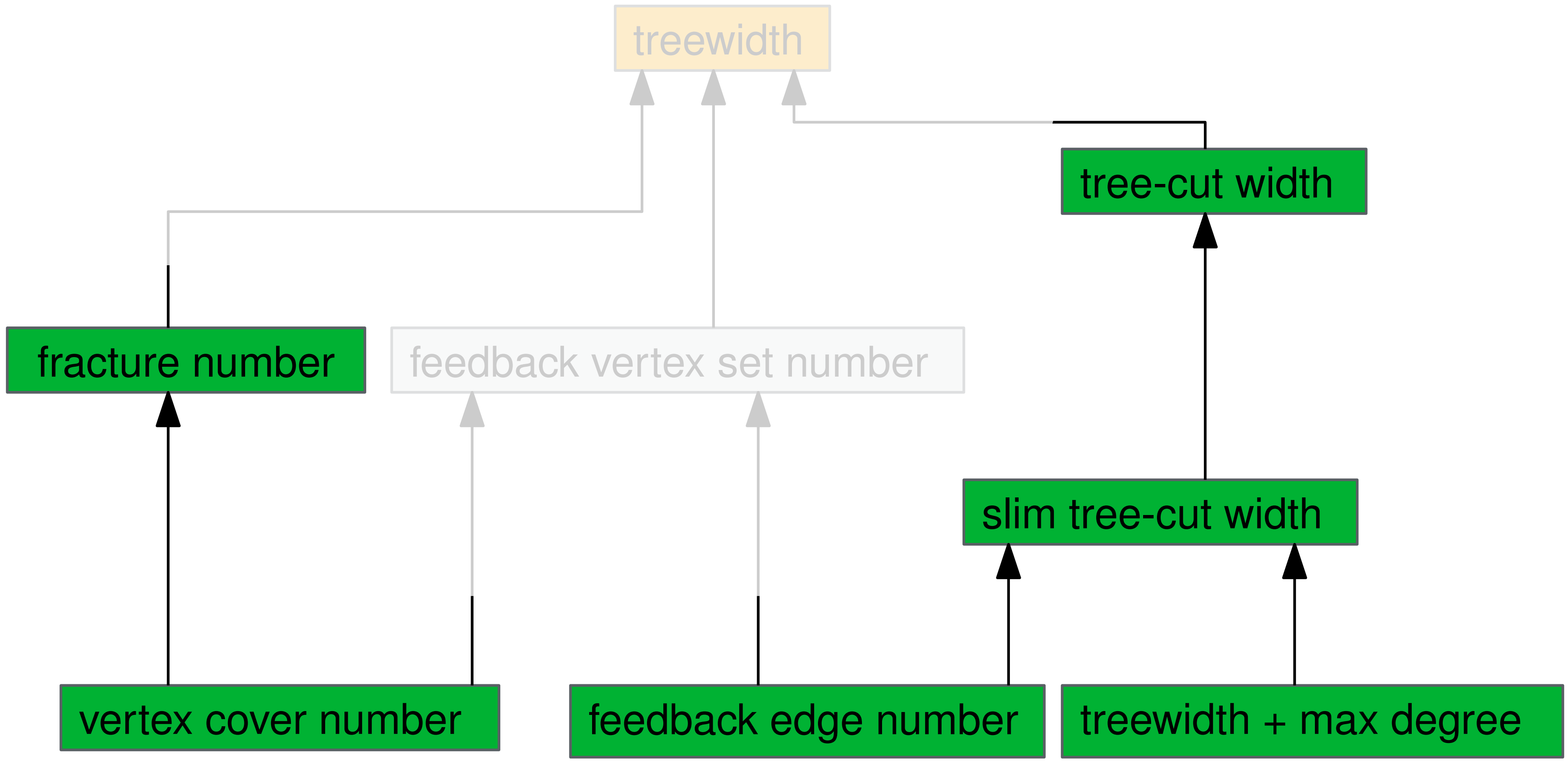


STP by Input Graph



Our Results:

[Aazami, Cheriyan, Jampani '12; Bodlaender et al. '23]



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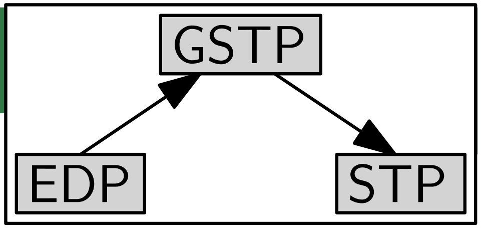
Recap: Our Contribution



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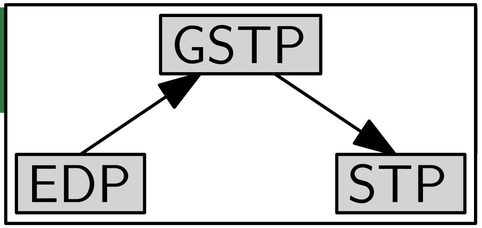

- Generalize the notion of the augmented graph to GSTP

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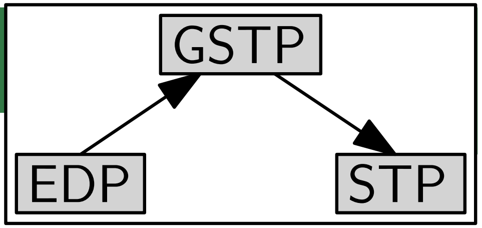

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- Generalize all known structural FPT algorithms for EDP to GSTP

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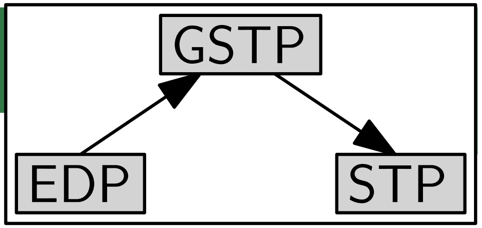

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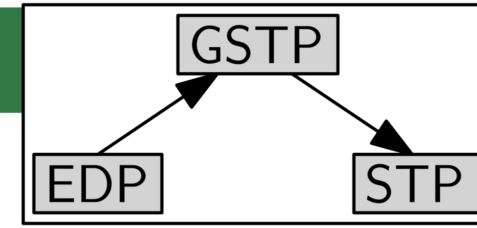

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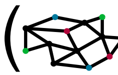
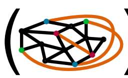
  Results for STP by input graph () coincide with GSTP by augmented graph ().

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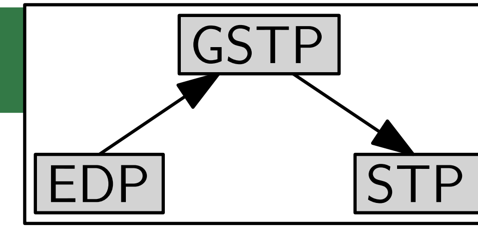

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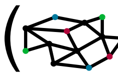
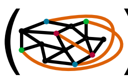
  Results for STP by input graph () coincide with GSTP by augmented graph ().
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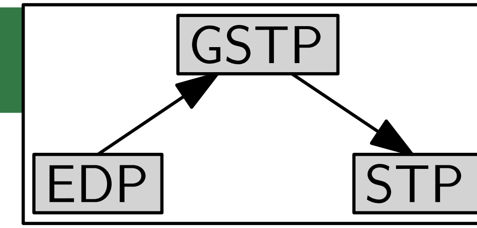

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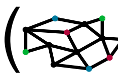
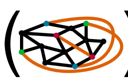
  Results for STP by input graph () coincide with GSTP by augmented graph ().
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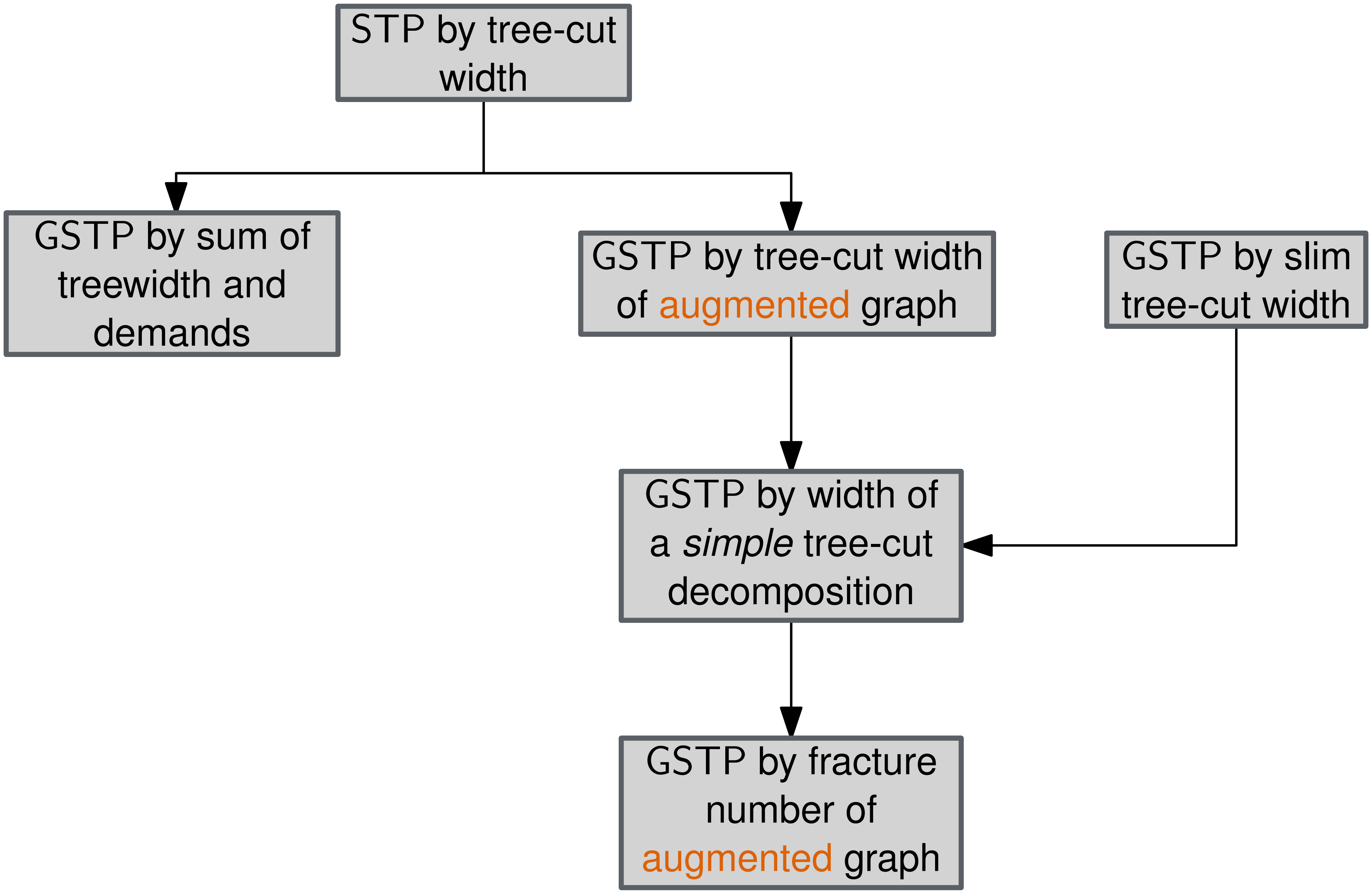
Thanks for your attention

Our Methods


STP by tree-cut width
GSTP by sum of treewidth and demands
GSTP by tree-cut width of augmented graph
GSTP by slim tree-cut width
GSTP by width of a *simple* tree-cut decomposition
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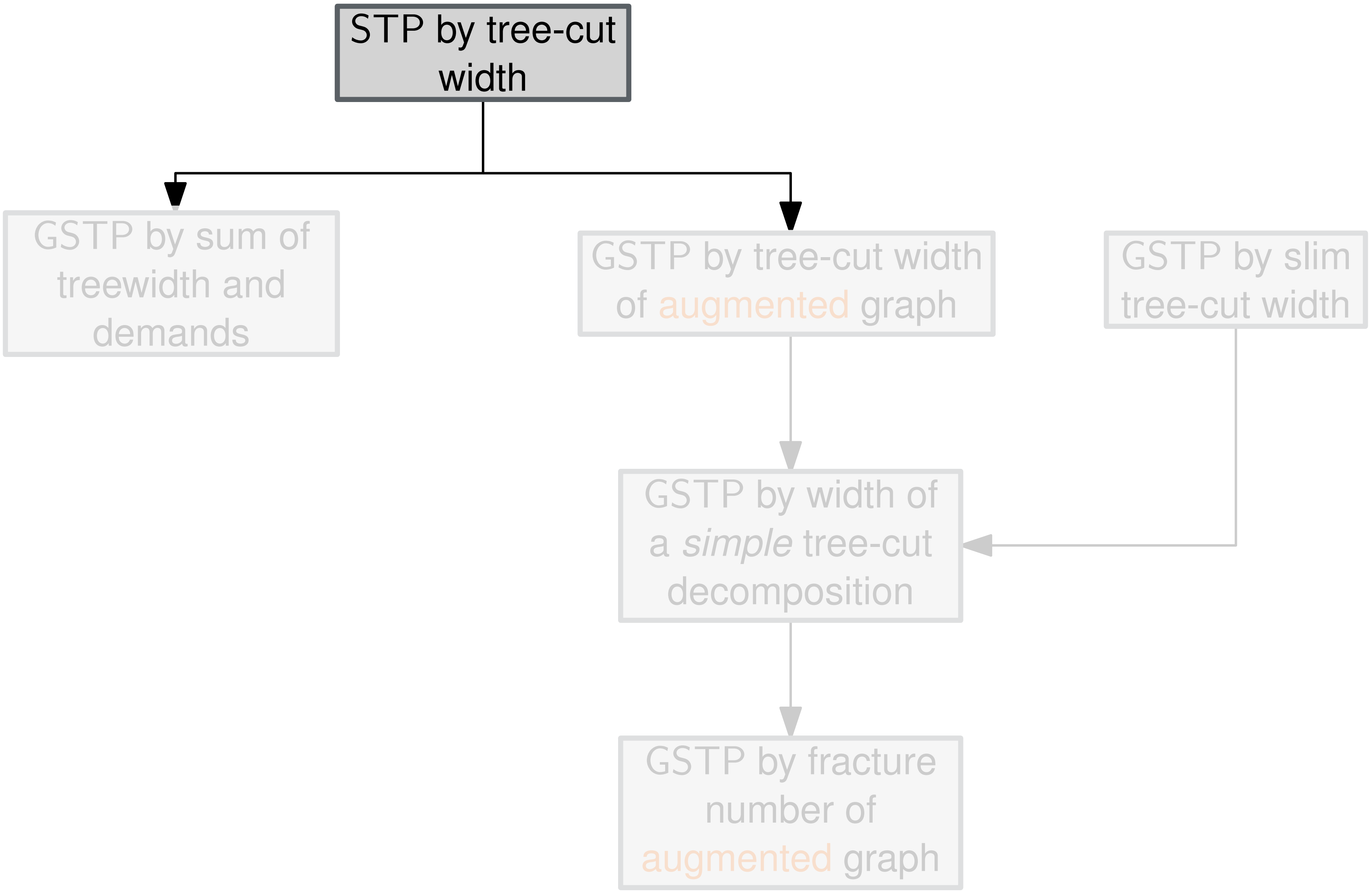
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Win-Win argument

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Solve using algorithm for GSTP by augmented tree-cut width.

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Otherwise ($|T| > w$):

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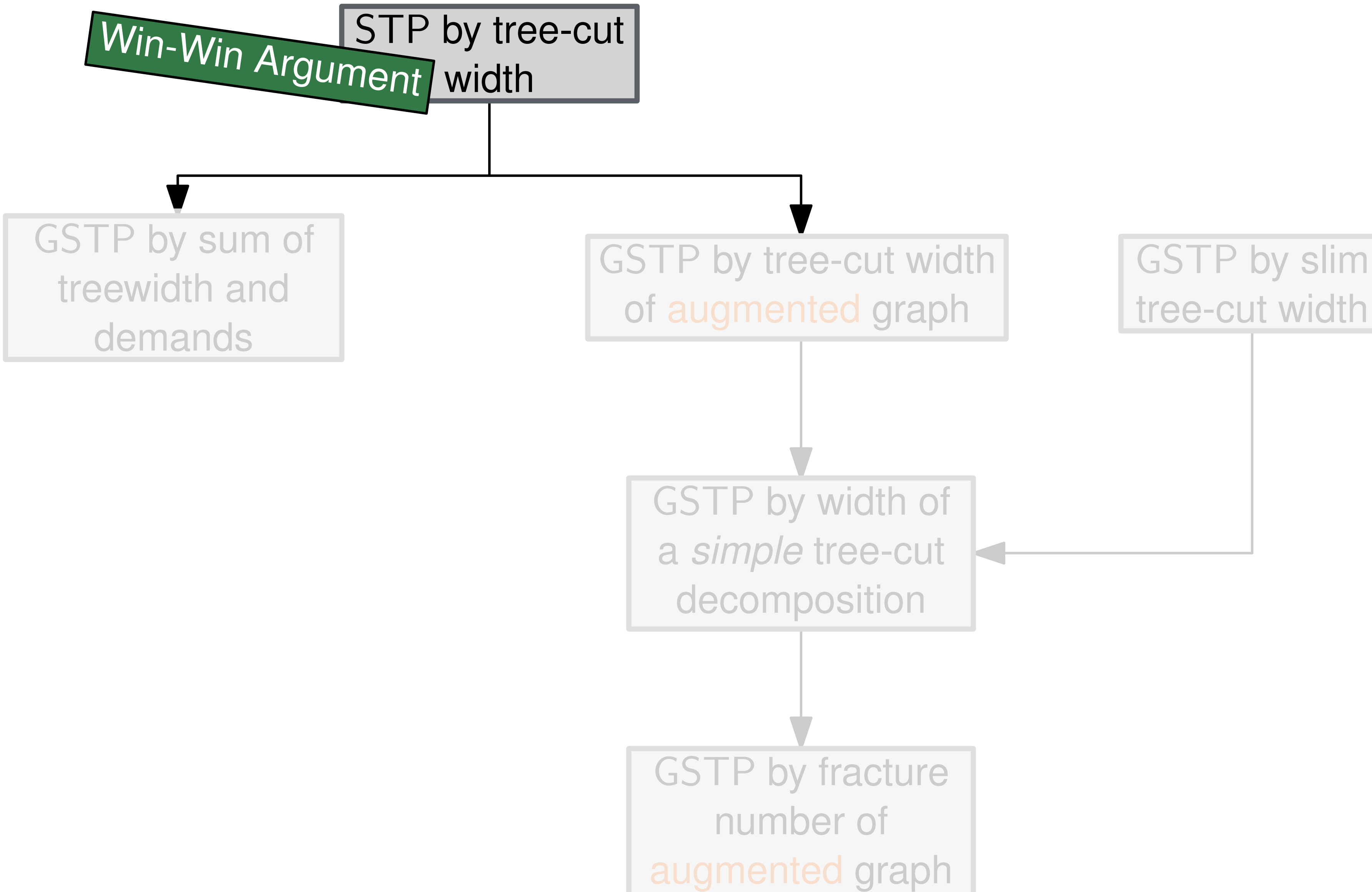
  - $\rightarrow T$ is at most w-edge connected.
- If $d > w$: Abort!
- Otherwise ($d \leq w$): solve using dynamic program w.r.t. sum of tree width and demand.

Our Methods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