Relative Compressed Reverse Suffix Array

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joint work with
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Suffixes

- Given a text $T[1, n]$, the suffix beginning at position j is the substring $T[j, n]$.
- **Example:** For Text $T =$ "BANANA$"

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2. ANANA$
3. NANA$
4. ANA$
5. NA$
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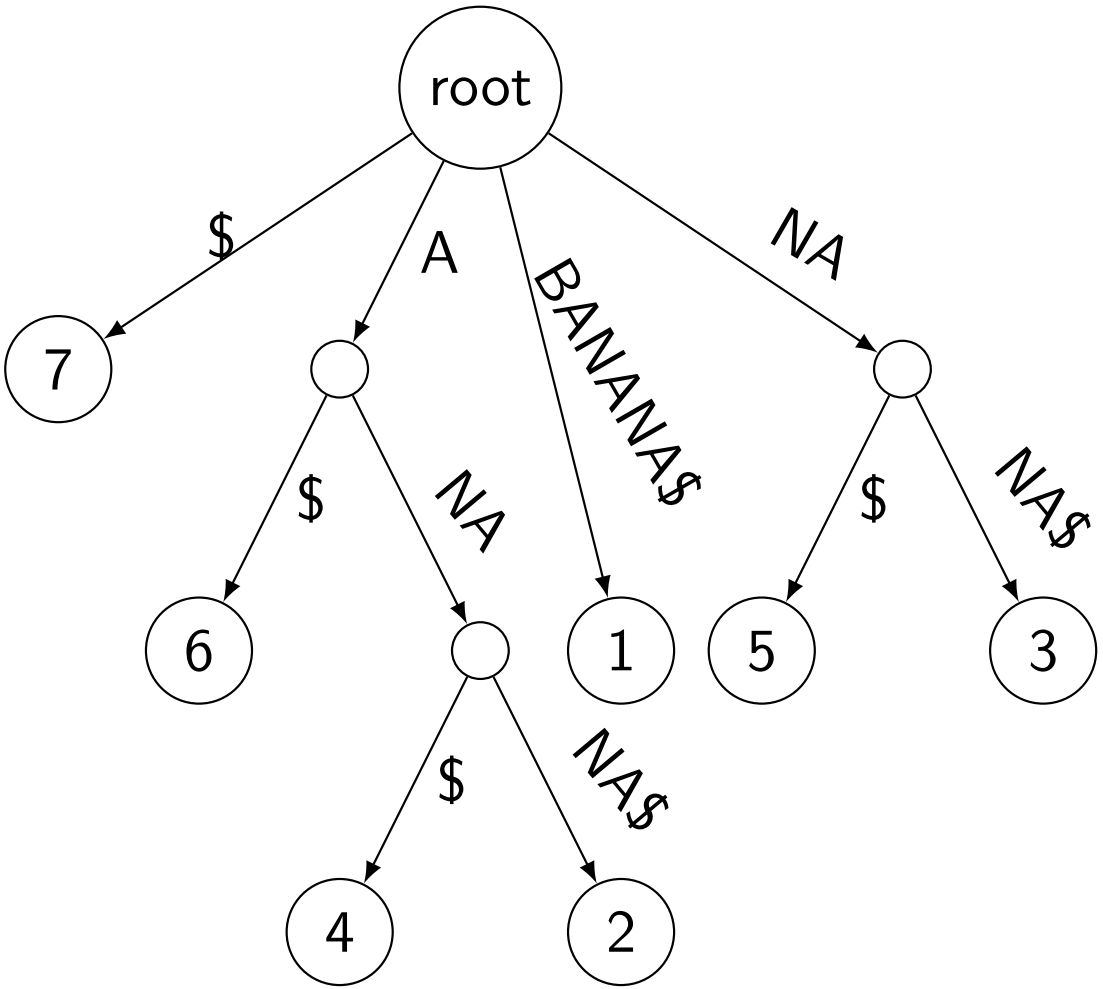
Suffix Trees



- Leaves are suffixes of T in lexicographic order (in the left-to-right leaf order).
- Internal nodes capture common prefixes, and their string depth equals that common-prefix length.

Figure: Suffix Tree of T = "BANANA$"

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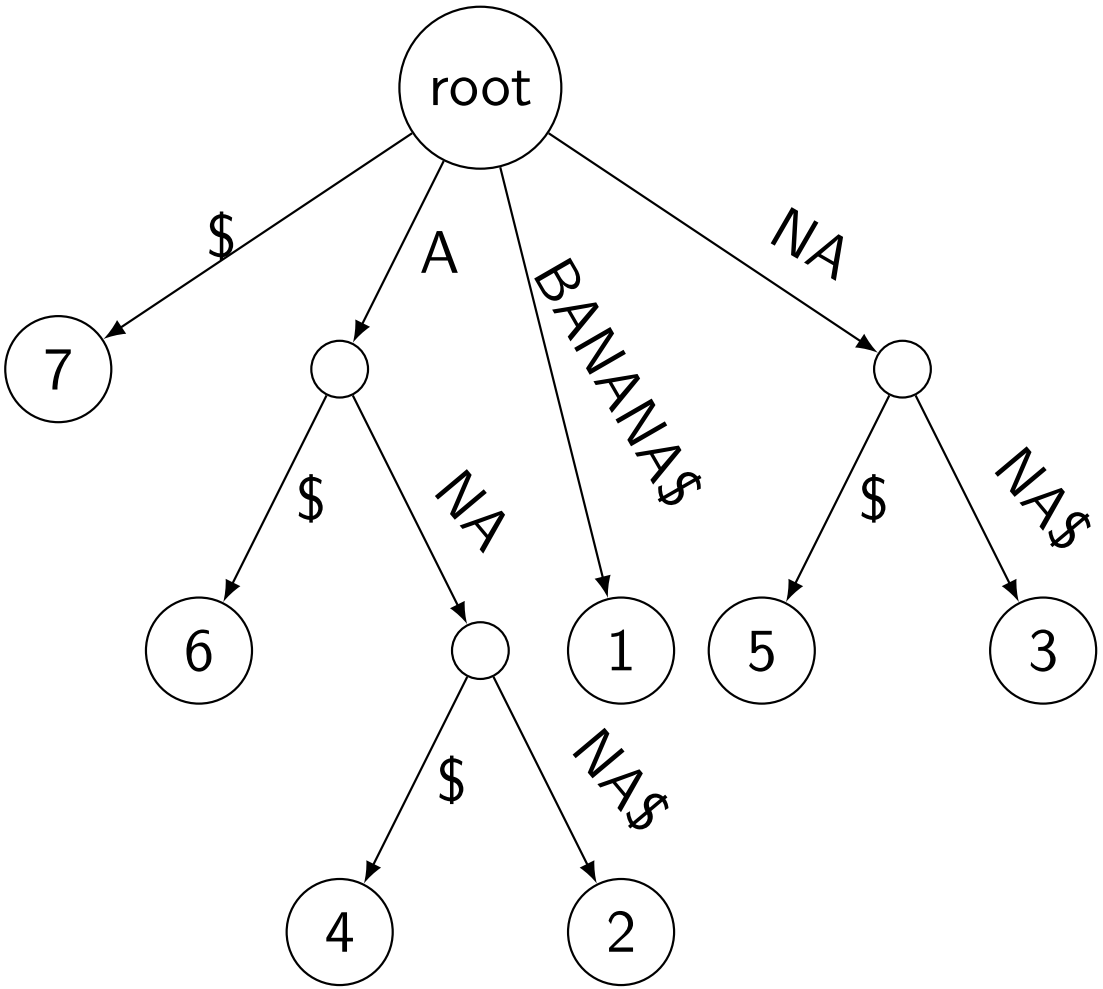

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SA, ISA, LCP, and BWT

- $\mathrm{SA}[i]$: i^{th} lexicographically smallest suffix.
- $\mathrm{ISA}[i]$: lexicographic rank of suffix $T[i \ldots n]$.
- $\mathrm{LCP}[i]$: length of the longest common prefix of i^{th} and $(i+1)^{th}$ lexicographically smallest suffixes.
- $\mathrm{BWT}[i] := T[(\mathrm{SA}[i]-1)(mod\ n)]$ (Burrows-Wheeler Transform)

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Store SA of $\overleftarrow{\mathrm{T}}$ explicitly (Trivial)	$n \log n$ bits	$\mathcal{O}(1)$ (direct access)
Compressed Suffix Array [1] / FM-index [2] of $\overleftarrow{\mathrm{T}}$	$n \log \sigma +$ l.o.t bits	$\mathcal{O}(\log n)$

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SA queries on $\overleftarrow{\mathsf{T}}$ can be supported in time $t_{\mathrm{sa}} \cdot \mathcal{O}(\log^* n)$ with an auxiliary structure of $\mathcal{O}(n)$ bits of space.

Theorem 2

SA queries on $\overleftarrow{\mathsf{T}}$ can be supported with $\mathcal{O}(n/\kappa)$ bits and a query time $t_{\mathrm{sa}} \cdot \mathcal{O}((\kappa \log^* n + \kappa^3 \log \kappa) \cdot \log^2 \kappa)$ where κ is a parameter.

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pLCP data structure

- Recall $\mathrm{LCP}[i]$ is the length of longest common prefix of i^{th} and $(i+1)^{th}$ lexicographically smallest suffixes.
- pLCP array is nothing but LCP array stored in text order.
- From Sadakane's result [3], pLCP can be stored in $2n + o(n)$ bits and queried in $\mathcal{O}(1)$ time.
- Similar to pLCP, one can also define pLCS which is LCS (Longest Common Suffix) stored in text order.

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Generalizing pLCP: the pLCP_π family

- For an integer $\pi \neq 0$, define $\mathrm{LCP}_\pi[i]$ as the length of longest common prefix of i^{th} and $(i+\pi)^{th}$ lexicographically smallest suffixes.
- Formally $\mathrm{LCP}_\pi[i] = \mathrm{LCE}(\mathrm{SA}[i], \mathrm{SA}[i+\pi])$
- Naturally, $\mathrm{pLCP}_\pi[j]$ is nothing but LCP_π stored in text order.
- The same encoding by Sadakane still holds. Therefore, for each fixed π, the pLCP_π can be encoded in $\mathcal{O}(n)$ bits and $\mathcal{O}(1)$-time access.
- Analogously, we also have pLCS_π structures.

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Quick Remark

- A suffix of $\overleftarrow{\mathsf{T}}$ beginning at position j is the reverse of prefix of T ending at $(n - j + 1)$.
- Define $\mathrm{PA}[i]$ as the i^{th} lexicographically smallest reverse prefix of T.
- So, we focus on answering the equivalent PA query when asked for a $\mathrm{SA}_{\overleftarrow{\mathsf{T}}}[i]$ query.

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Algorithm: High-level description

- Sample leaves of prefix tree (PT) of T in $\log^* n$ levels. (i.e., PA_h arrays for $h \in [\log^* n]$ where PA_h has $\frac{n}{\Delta_h}$ entries for Δ_h being the sampling factor).

- Levels are chosen in such a way that any leaf i is sampled in some level $f \in [\log^* n]$.

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- From Level 1, we decode entries of Level 2, 3, .., until f, finally retrieving $\mathrm{PA}[i]$.
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- Overall time taken is at most $\log^*(n)\times$ (Time to decode a leaf of one level from its closest leaves sampled in the previous level).

Algorithm: High-level description (Contd.)

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- **Succinct topologies** of the suffix tree of T and the prefix tree of T.
- **Sampling in levels.** For $h \in [1, \log^* n]$, define $\Delta_h = 2^{\lceil 2 \log^{(h+1)} n \rceil}$, i.e. the next power of 2 above $(\log^{(h)} n)^2$. Also $\Delta_0 = n + 1$.
- **Level 1:** if $\Delta_1 \mid i$, store $\mathrm{PA}_1[i] = \mathrm{PA}[i]$ explicitly.
- **Higher levels:** if $\Delta_h \mid i$ but $\Delta_{h-1} \nmid i$, store only a small offset θ_i $(\leq \Delta_{h-1})$ in $\mathrm{PA}_h[i]$.
- **Auxiliary sampled pLCP_π and pLCS_π structures:** For each $h \in [1, \log^* n]$, we maintain $2\frac{\Delta_{h-1}}{\Delta_h}$ pLCP_π and pLCS_π structures (each structure maintaining only $\frac{n}{\Delta_{h-1}}$ entries), which will be helpful during the recursive decode.
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Decoding a level-2 node from level-1 values (Contd.)

- Say we want to decode $\mathrm{PA}[i]$ where i^{th} leaf is sampled in level-2 (i.e., $\Delta_1 \nmid i$, but $\Delta_1 \mid i$).
- Find the closest level-1 sampled leaves to left and right of i (i.e., $i' = \Delta_1 \lfloor \frac{i}{\Delta_1} \rfloor$ and $i'' = \Delta_1 \lceil \frac{i}{\Delta_1} \rceil$).
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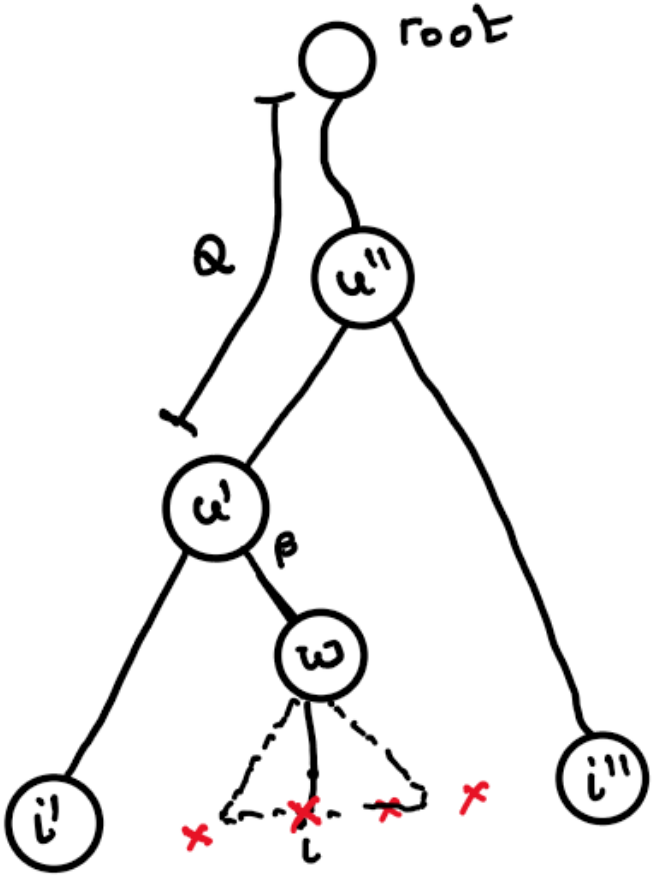

Figure: Prefix Tree of T

1. Find $\overleftarrow{\mathrm{Q}}$ (using $\mathrm{PA}[i']$ and pLCP_π structures) - $\mathcal{O}(1)$ time
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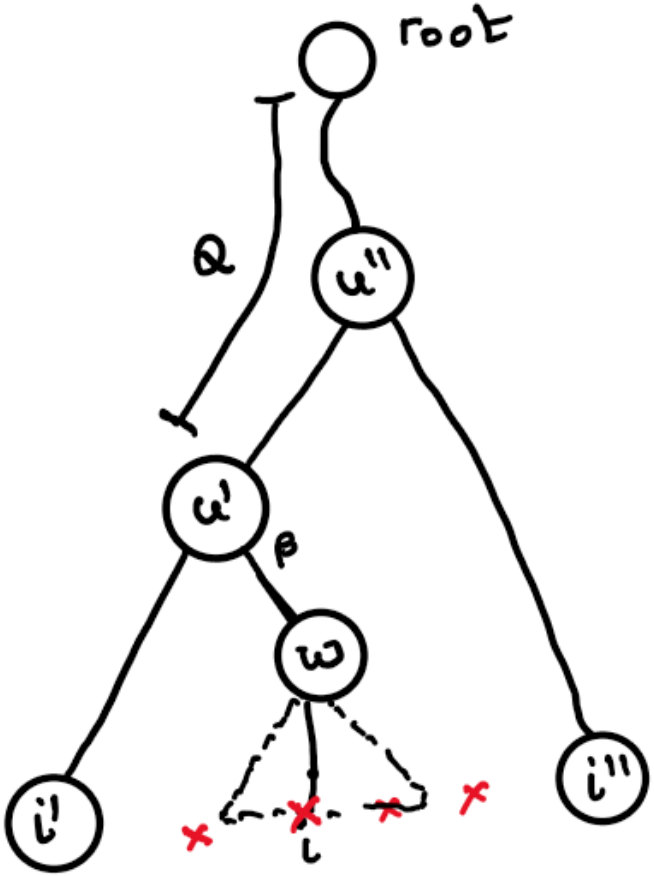

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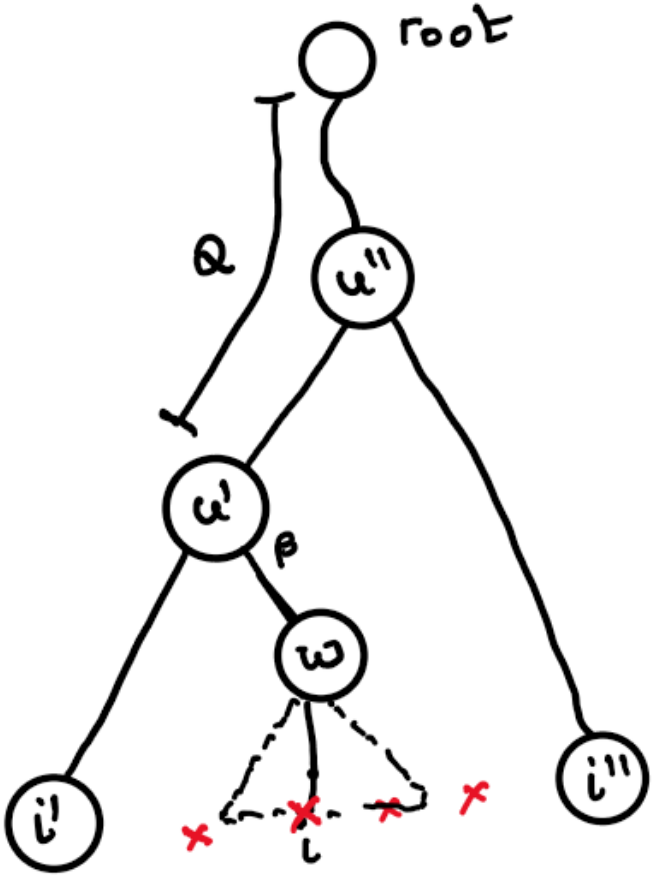

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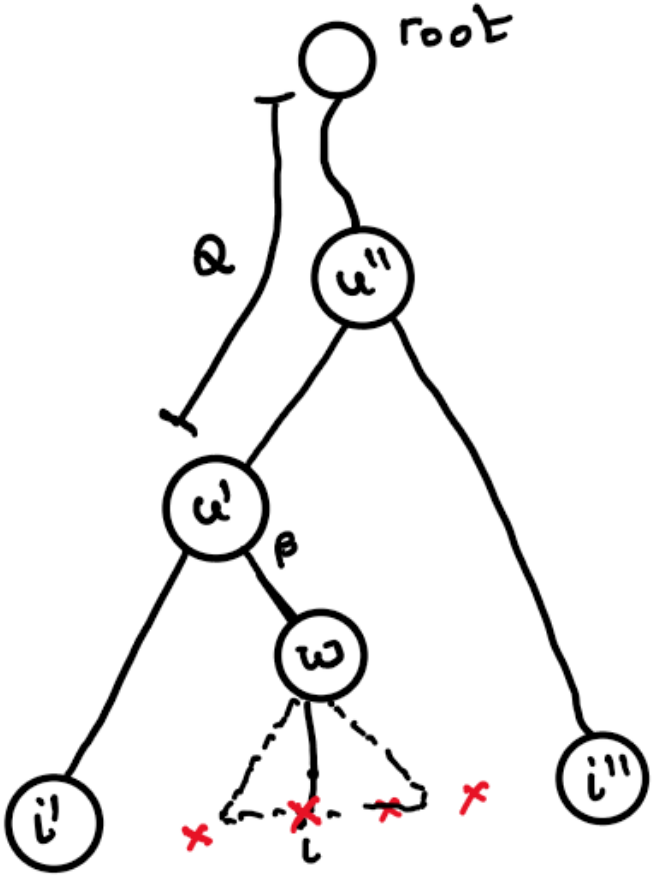

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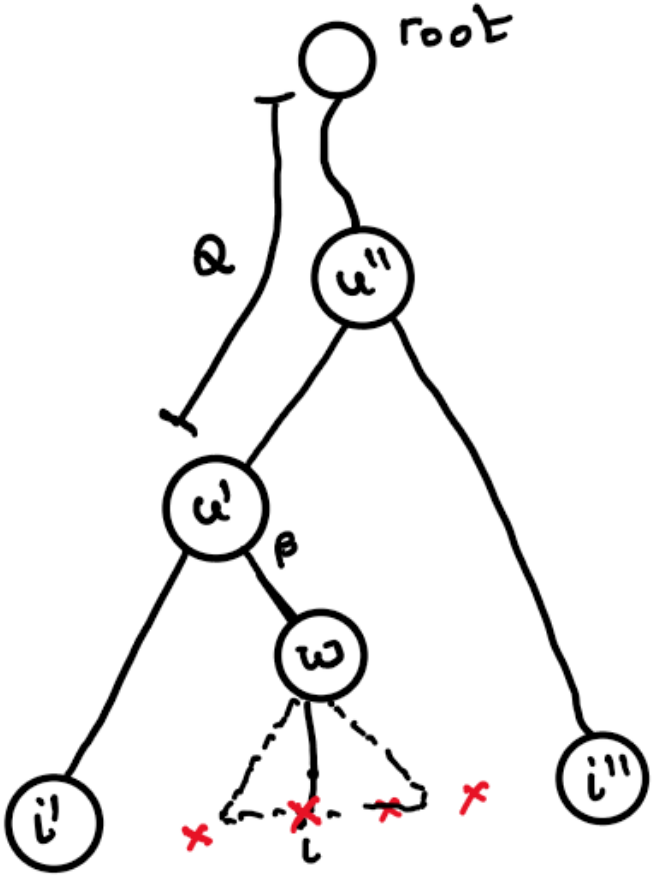

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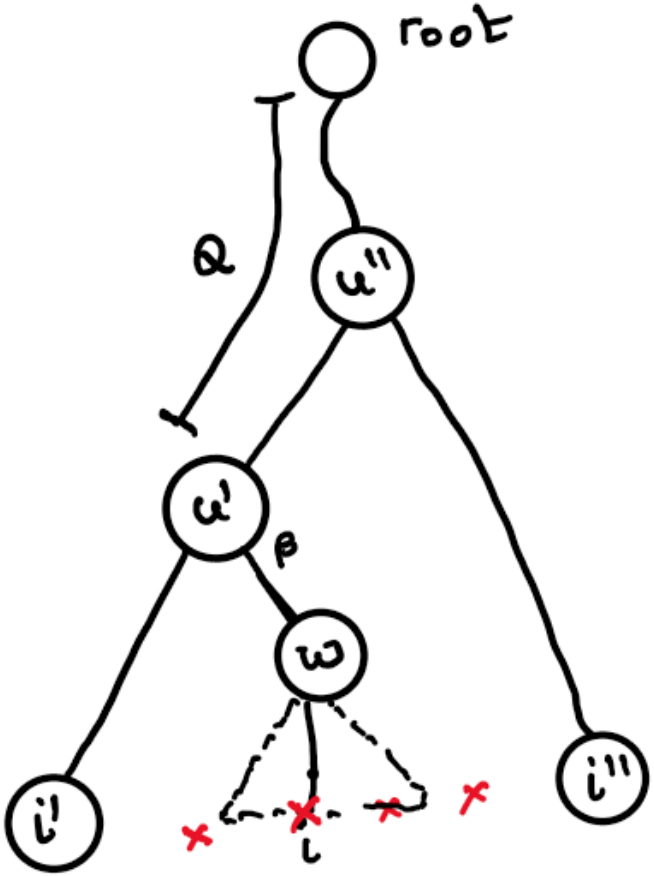

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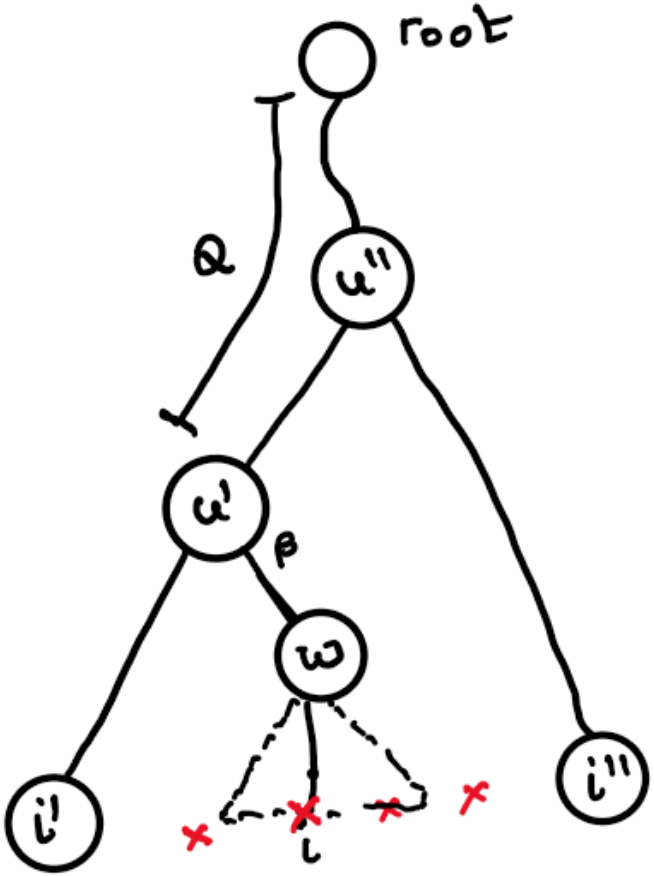

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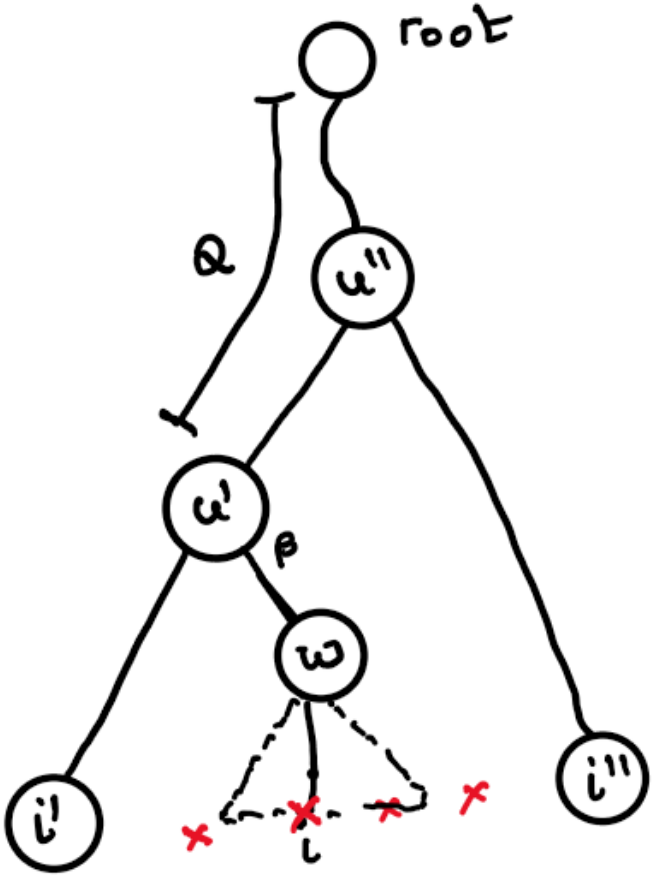

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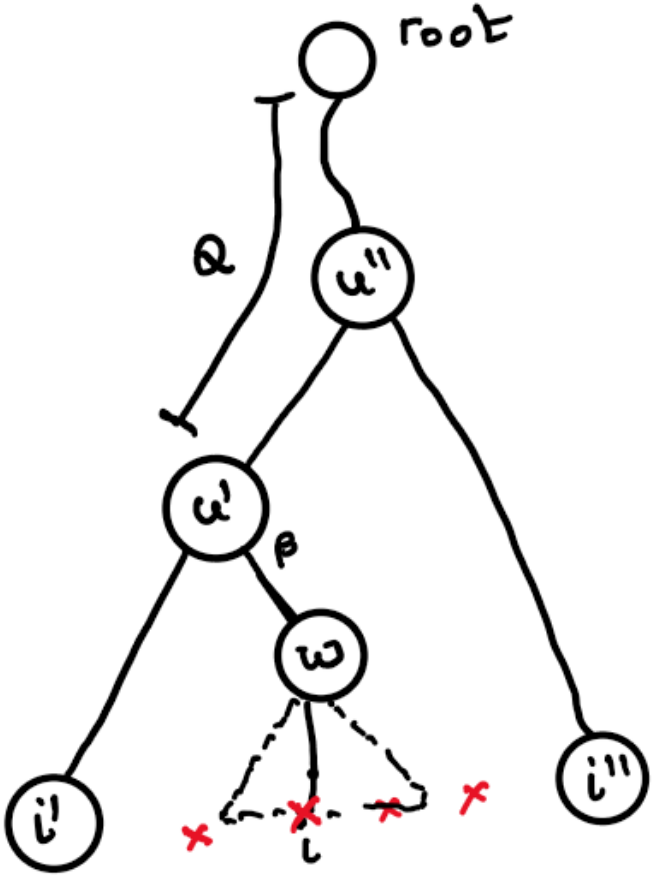

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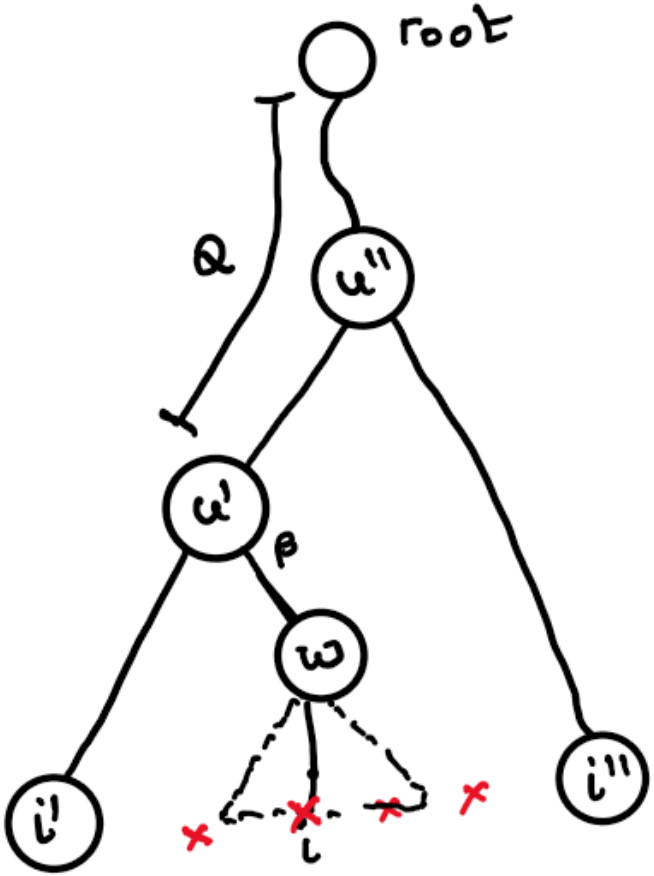

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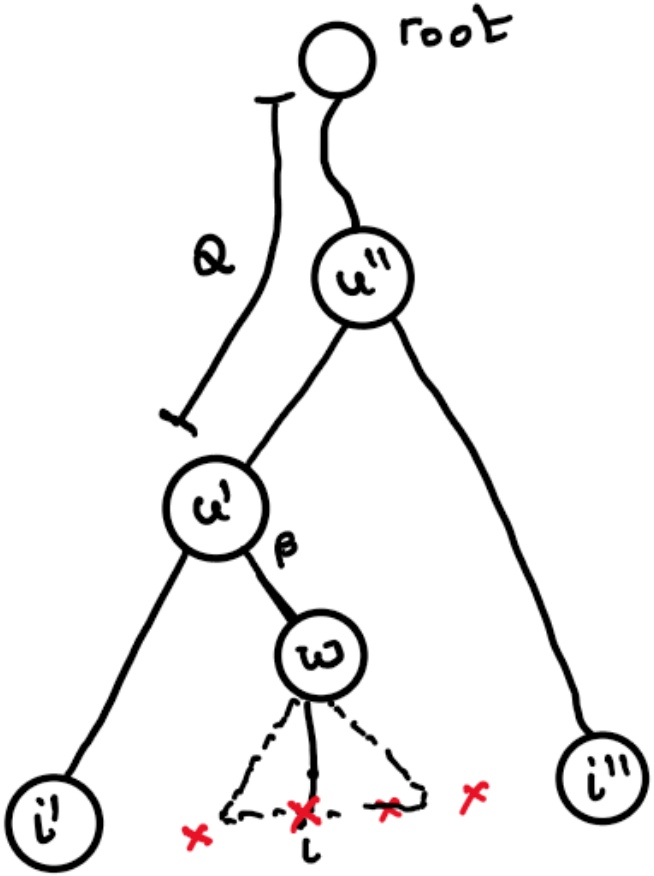

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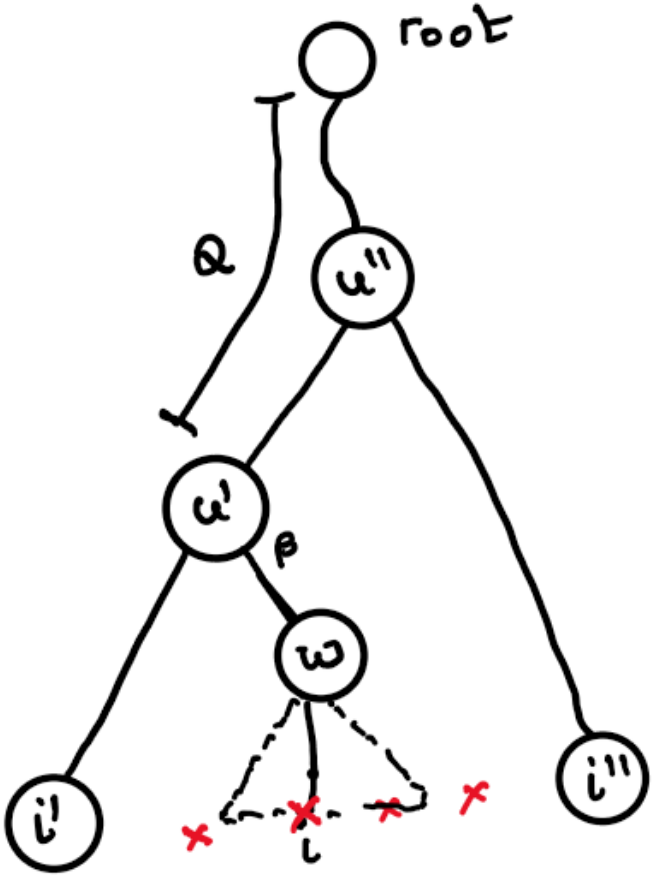

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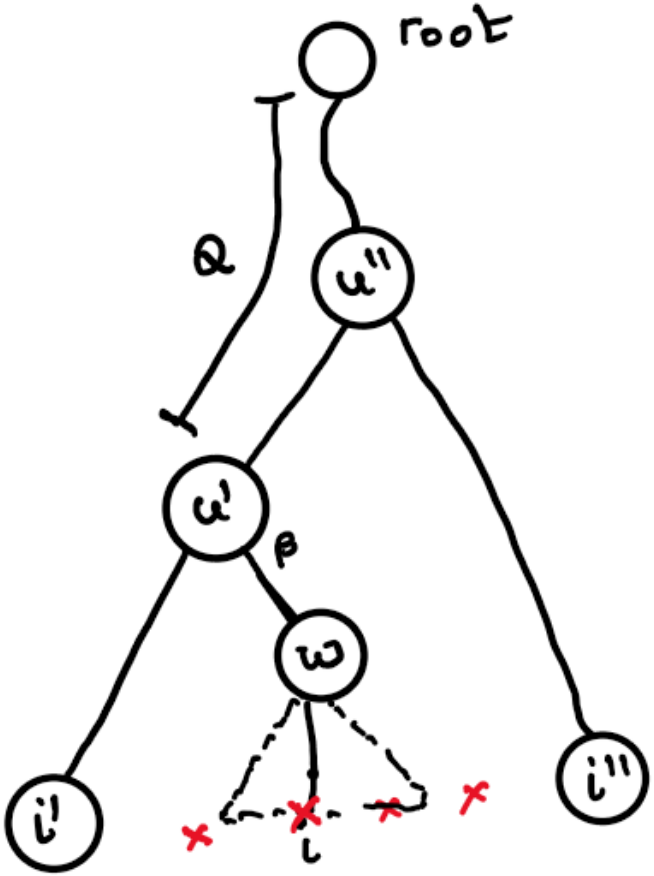

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5. Compute $\mathrm{PA}[i]$ using the offset θ_i - $\mathcal{O}(t_{\mathrm{sa}})$ time
6. Total - $\mathcal{O}(t_{\mathrm{sa}} + \log n + t_{\mathrm{quantile}} + t_{\mathrm{wt}}) \in \mathcal{O}(t_{\mathrm{sa}})$ time.

Decoding a level-2 node from level-1 values (Contd.)

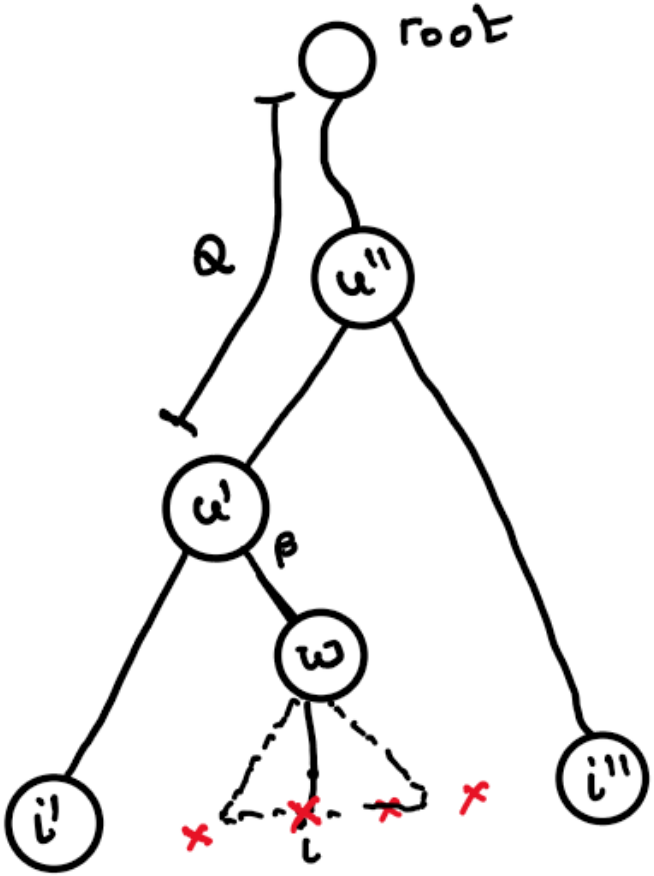

Figure: Prefix Tree of T

1. Find $\overleftarrow{\mathrm{Q}}$ (using $\mathrm{PA}[i']$ and pLCP_π structures) - $\mathcal{O}(1)$ time
2. Find the suffix range of $\overleftarrow{\mathrm{Q}}$ - $\mathcal{O}(t_{\mathrm{sa}} + \log n)$ time
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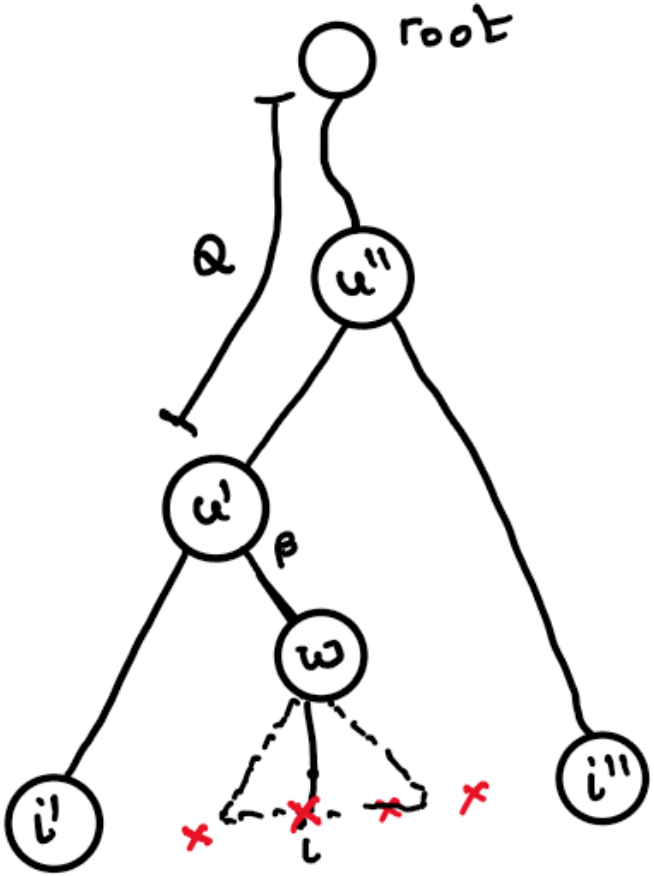

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Concluding First approach

- Decoding a level-2 sampled leaf from level-1 takes $\mathcal{O}(t_{\text{sa}})$ time.
- Since every query index belongs to one of the $\log^* n$ levels, we repeat this inductively for at most $\log^* n$ iterations.
- Overall algorithm has a $\mathcal{O}(t_{\text{sa}} \cdot \log^*(n))$ runtime.

Theorem 1

SA queries on $\overleftarrow{\mathrm{T}}$ can be supported in time $t_{\text{sa}} \cdot \mathcal{O}(\log^* n)$ with an auxiliary structure of $\mathcal{O}(n)$ bits of space.

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Thank You!

Any Questions?

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