Pumping-Like Results for Copyless Cost Register Automata and Polynomially Ambiguous Weighted Automata

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Our models

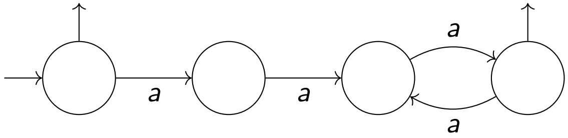
What is a polynomially ambiguous weighted automaton?
What is a copyless cost register automaton?

Our models

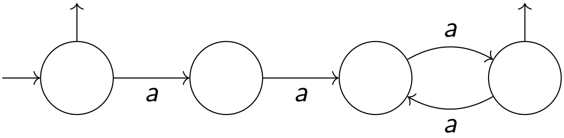
What is a polynomially ambiguous weighted automaton?
What is a copyless cost register automaton?

Is every context-free language regular?

Single letter alphabet

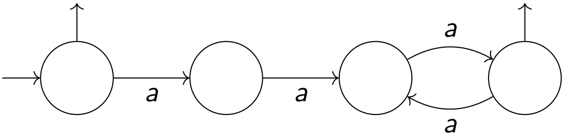


Single letter alphabet



$$a_n := a^n \stackrel{?}{\in} L$$

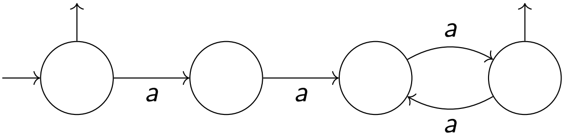
Single letter alphabet



$$a_n := a^n \overset{?}{\in} L$$

$$\begin{array}{c}\varepsilon \\ \downarrow \\ \top\end{array}$$

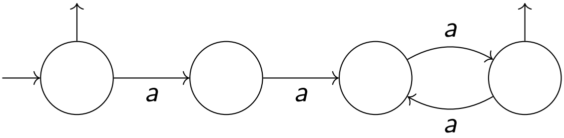
Single letter alphabet



$$a_n := a^n \overset{?}{\in} L$$

$\varepsilon \downarrow \top$ $\qquad$ $a \downarrow \bot$

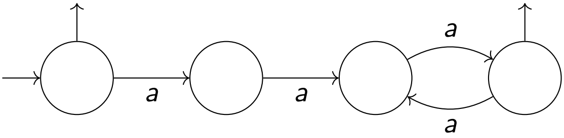
Single letter alphabet



$$a_n := a^n \overset{?}{\in} L$$

$$\begin{matrix} \varepsilon & a & a^2 \\ \downarrow & \downarrow & \downarrow \\ \top & \bot & \bot \end{matrix}$$

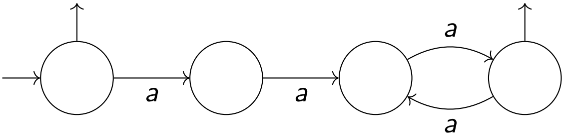
Single letter alphabet



$$a_n := a^n \overset{?}{\in} L$$

ε	a	a^2	a^3
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\top$	$\bot$	$\bot$	$\top$

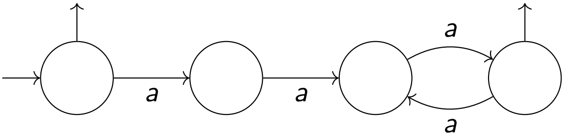
Single letter alphabet



$$a_n := a^n \stackrel{?}{\in} L$$

ε	a	a^2	a^3	a^4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\top$	$\bot$	$\bot$	$\top$	$\bot$

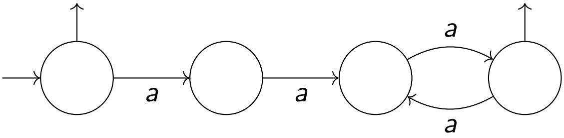
Single letter alphabet



$$a_n := a^n \stackrel{?}{\in} L$$

ε	a	a^2	a^3	a^4	a^5
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$

Single letter alphabet



$$a_n := a^n \overset{?}{\in} L$$

$$\begin{matrix} \varepsilon & a & a^2 & a^3 & a^4 & a^5 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \cdots \\ \top & \bot & \bot & \top & \bot & \top & \end{matrix}$$

$$a_n := uw^n v \overset{?}{\in} L$$

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Even length

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$u = \varepsilon, w = a, v = \varepsilon$

$$\begin{array}{cccc} \varepsilon & a & a^2 & a^3 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \top & \bot & \top & \bot \end{array}$$

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$$u = \varepsilon, w = aa, v = \varepsilon$$

$$\begin{array}{cccc} \varepsilon & a^2 & a^4 & a^6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \top & \top & \top & \top \end{array}$$

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$u = a, w = aa, v = \varepsilon$

$$\begin{array}{cccc} a & a^3 & a^5 & a^7 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \bot & \bot & \bot & \bot \end{array}$$

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Even length

$\{a^n b^n | n \in \mathbb{N}\}$

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$$\begin{array}{cccc} a & a^2 & a^3 & a^4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \bot & \top & \bot & \top \end{array}$$

$\{a^n b^n | n \in \mathbb{N}\}$

$u = a^k, w = b, v = \varepsilon$

$$a_n := uw^n v \overset{?}{\in} L$$

Even length

$u = \varepsilon, w = a, v = \varepsilon$

$$\begin{array}{cccc} \varepsilon & a & a^2 & a^3 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \top & \bot & \top & \bot \end{array}$$

$u = \varepsilon, w = aa, v = \varepsilon$

$$\begin{array}{cccc} \varepsilon & a^2 & a^4 & a^6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \top & \top & \top & \top \end{array}$$

$u = a, w = aa, v = \varepsilon$

$$\begin{array}{cccc} a & a^3 & a^5 & a^7 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \bot & \bot & \bot & \bot \end{array}$$

$u = a, w = a, v = \varepsilon$

$$\begin{array}{cccc} a & a^2 & a^3 & a^4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \bot & \top & \bot & \top \end{array}$$

$\{a^n b^n | n \in \mathbb{N}\}$

$u = a^k, w = b, v = \varepsilon$

$$\begin{array}{ccccccc} a^k & a^k b & \dots & a^k b^{k-1} & a^k b^k & a^k b^{k+1} & \dots \\ \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \\ \bot & \bot & & \bot & \top & \bot & \end{array}$$

Summary

We have a model with simple single-letter behaviour, but we need more fine-grained analysis.
For a fixed automaton $\mathcal{A}$, look at the set of sequences

$$\left\{(a_n)_{n\in\mathbb{N}} = \mathcal{A}(uw^n v) \mid u, w, v \in \Sigma^*\right\}$$

Using the fact the automaton is fixed, look at limitations of this set.

What was known

Fact

For a weighted automaton $\mathcal{A}$ over K using a single letter alphabet we have that $\mathcal{A}(a^n)$ can be represented as $\sum p_i(n)\alpha_i^n$, where $\alpha_i \in \overline{K}$ and $p_i \in \overline{K}[x]$.

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Theorem (Barloy-Fijalkow-Lhote-Mazowiecki, 2020)

Over single letter alphabets, PAWA and CCRA are equivalent models.

Results

Theorem (Mazowiecki-P.-Smertnig, 2026)

For a function f recognised by a PAWA over K we have a finitely generated multiplicative subgroup $\Gamma \subseteq \overline{K}^*$ such that, for any u, w, v, if we have $f(uw^n v) = \sum p_k(n)\alpha_k^n$ then $\alpha_k \in \Gamma$.

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Theorem (Mazowiecki-P.-Smertnig, 2026)

For a function f recognised by a CCRA over K we have a finitely generated subring $R \subseteq K$ and a constant $m \in \mathbb{N}$ such that, for any u, w, v, if we have $f\left(uw^{m(n+1)}v\right) = \sum_i \left(n^i \sum_k c_{i,k}\alpha_{i,k}^n\right)$, then $\sum_k c_{i,k} \in R$.

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Thank you!

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