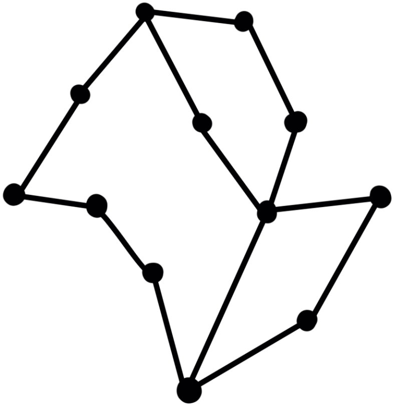
A polynomial bound on the pathwidth of graph edge-coverable by k shortest paths

Julien Baste[1] Lucas De Meyer[2] Ugo Giocanti[3]

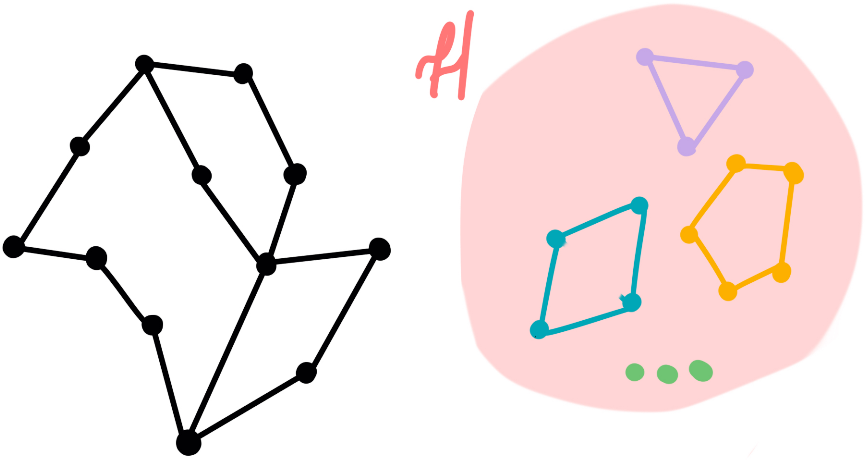
Étienne Objois[4] Timothé Picavet[5]

[1]Université de Lille, France
[2]LIRIS, Université Claude Bernard Lyon 1, France
[3]Jagellonian University, Krakow, Poland
[4]IRIF, Université Paris Cité, France
[5]LABRI, Université de Bordeaux, France

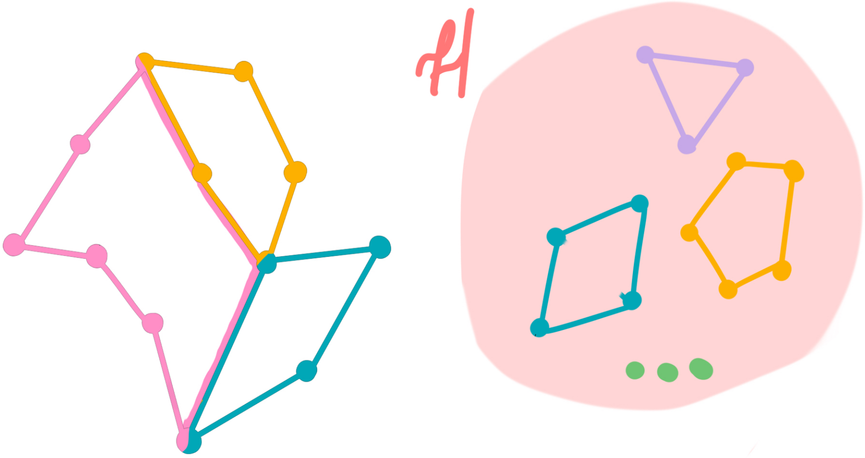
Isometric path edge-cover



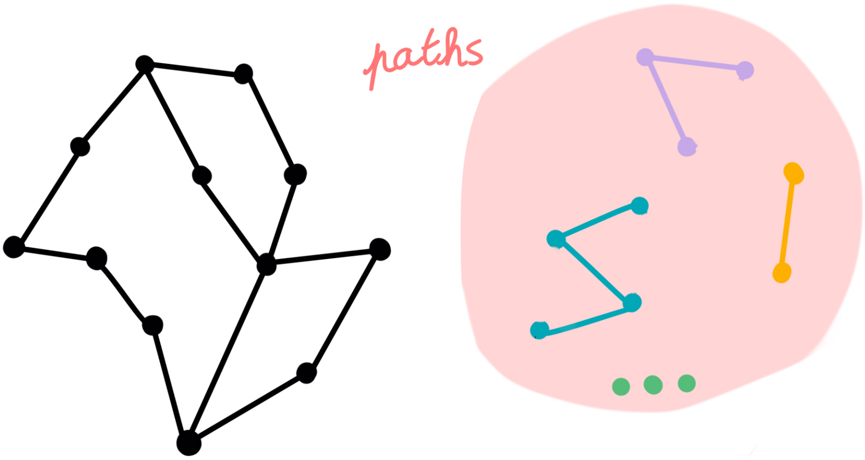
Isometric path edge-cover



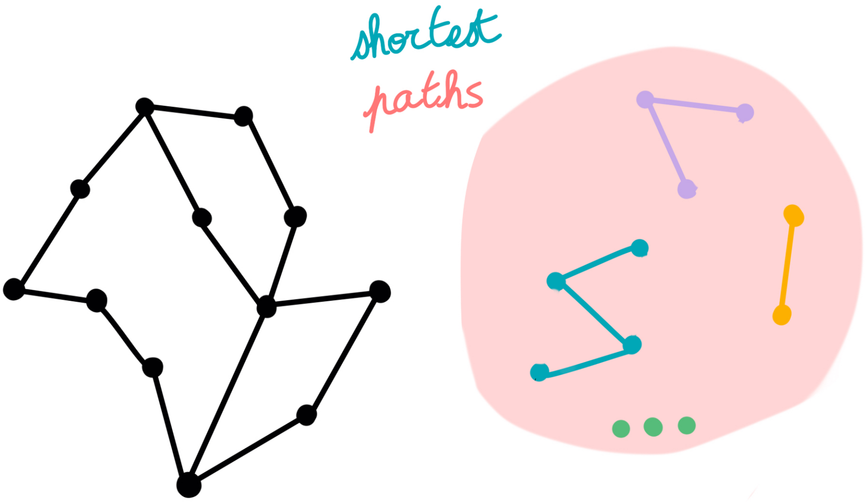
Isometric path edge-cover



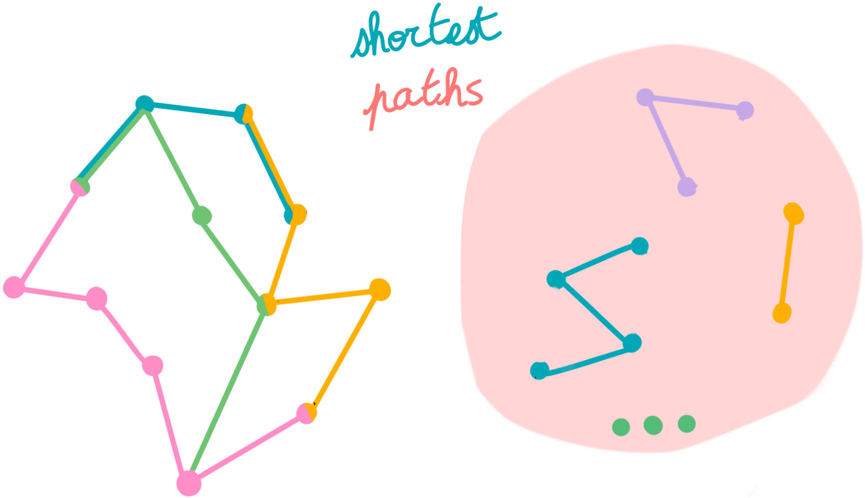
Isometric path edge-cover



Isometric path edge-cover



Isometric path edge-cover



Structure of graphs coverable by few paths

Isometric path edge-cover

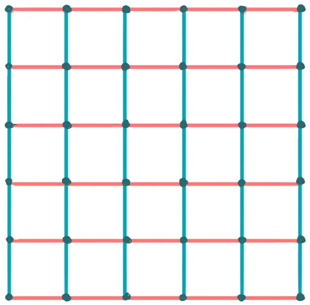
`In:` Graph G, integer k.

`Out:` Can the edges of G be covered by $\leq k$ shortest paths?

Question

G edge-coverable by a few shortest paths.

- *What is the structure of G?*
- *How close is G from a path?*
- *Can we bound the pathwidth of G?*



An edge-cover of the grid by shortest paths.

Structure of graphs coverable by few paths

Isometric path edge-cover

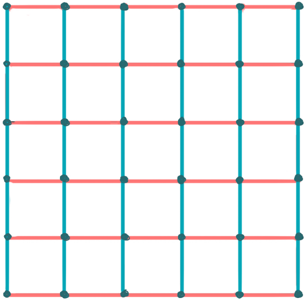
In: Graph G, integer k.

Out: Can the edges of G be covered by $\leq k$ shortest paths?

Question

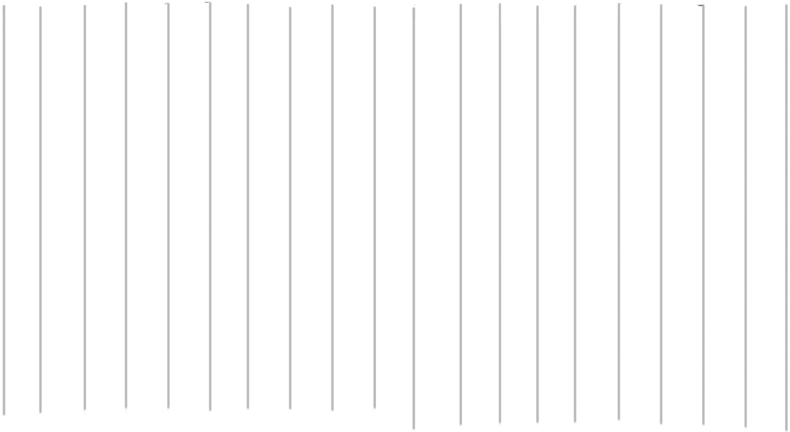
G edge-coverable by a few shortest paths.

- *What is the structure of G?*
- *How close is G from a path?*
- *Can we bound the pathwidth of G?*



An edge-cover of the grid by shortest paths.

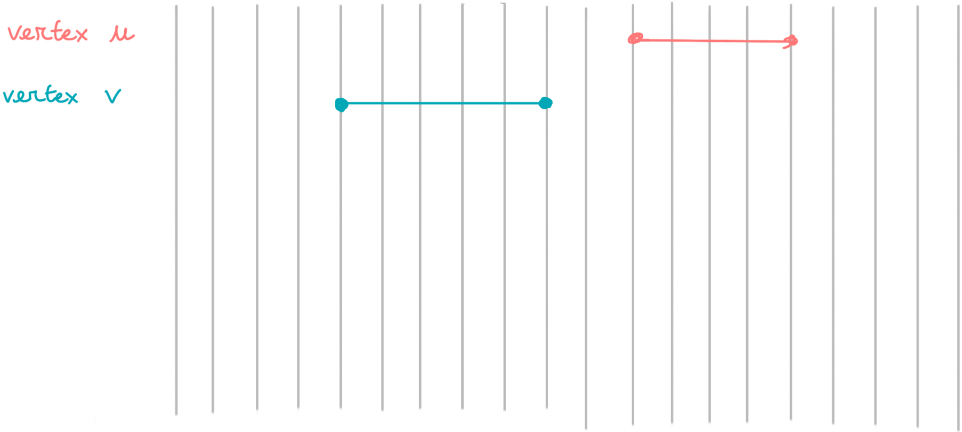
Path-decomposition $\mathcal{P}$ of a graph



pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p+1$

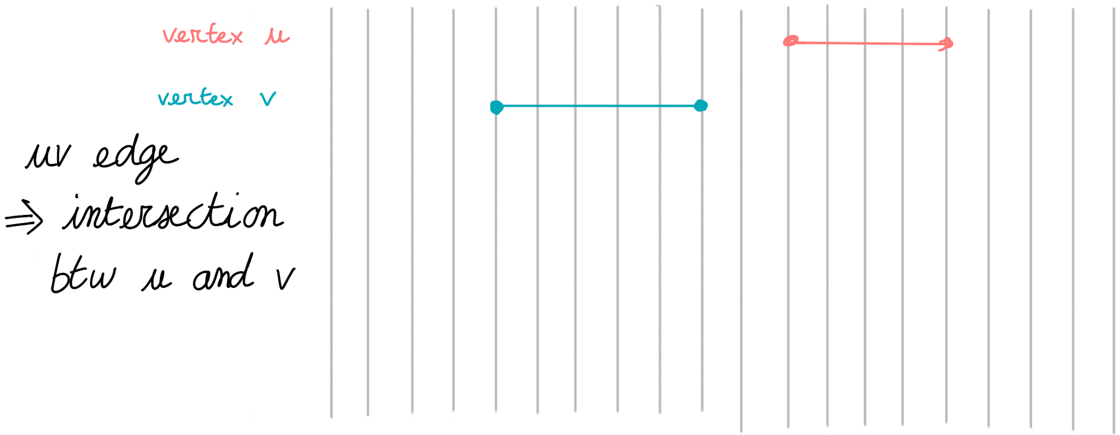
Pathwidth

Path-decomposition $\mathcal{P}$ of a graph



pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p + 1$

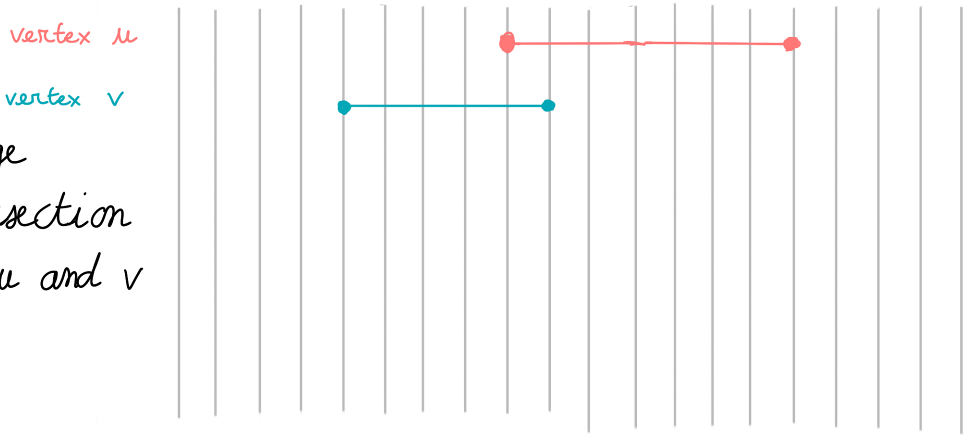
Path-decomposition $\mathcal{P}$ of a graph



pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p + 1$

Pathwidth

Path-decomposition $\mathcal{P}$ of a graph

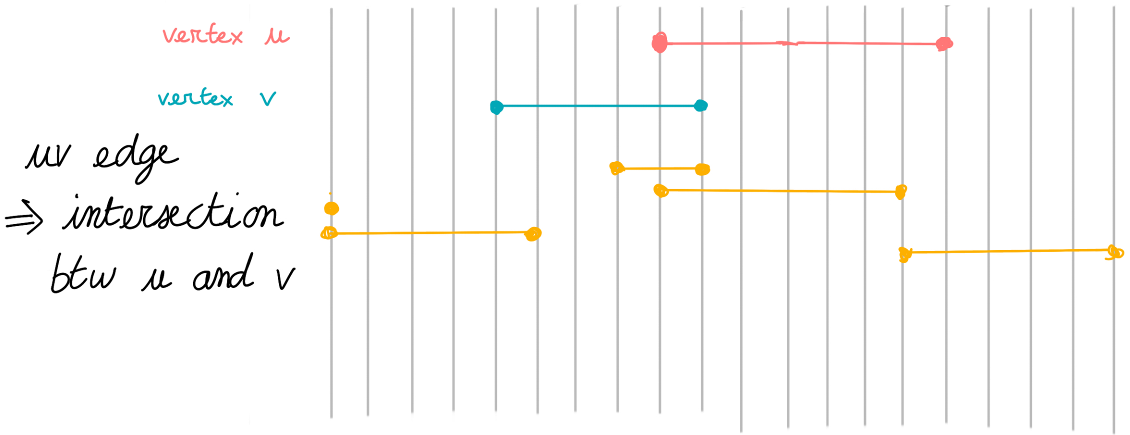


uv edge

⇒ intersection btw u and v

pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p + 1$

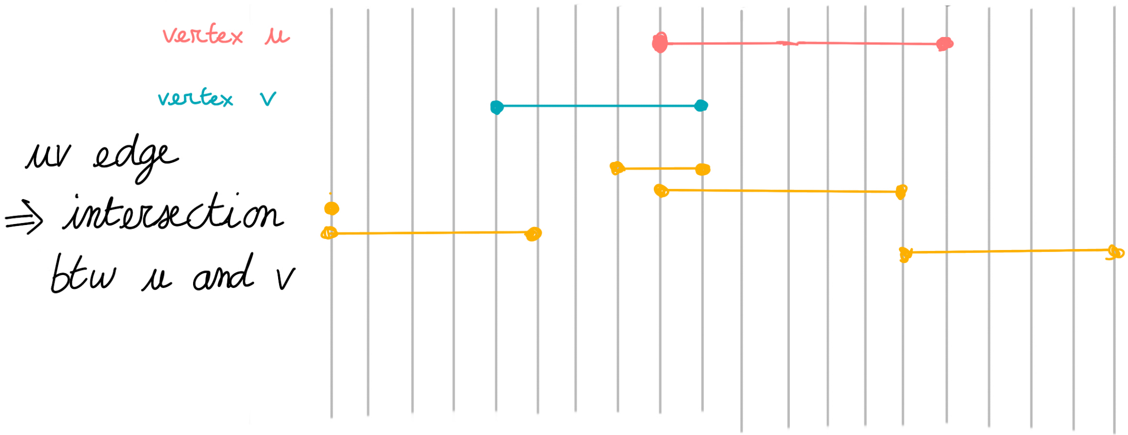
Path-decomposition $\mathcal{P}$ of a graph



pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p + 1$

Pathwidth

Path-decomposition $\mathcal{P}$ of a graph



pathwidth $\leq p$ = a path decomposition, where intersections have size $\leq p+1$

Pathwidth and path edge-covers

Graphs edge-coverable by a few shortest subpaths look like a path!

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(3^k)$.

Polynomial bounds?

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by 3 shortest paths. Then $\mathrm{pw}(G) \leq 3$.

Pathwidth and path edge-covers

Graphs edge-coverable by a few shortest subpaths look like a path!

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(3^k)$.

Polynomial bounds?

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by 3 shortest paths. Then $\mathrm{pw}(G) \leq 3$.

Pathwidth and path edge-covers

Graphs edge-coverable by a few shortest subpaths look like a path!

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(3^k)$.

Polynomial bounds?

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by 3 shortest paths. Then $\mathrm{pw}(G) \leq 3$.

Pathwidth and path edge-covers

Graphs edge-coverable by a few shortest subpaths look like a path!

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(3^k)$.

Polynomial bounds?

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by 3 shortest paths. Then $\mathrm{pw}(G) \leq 3$.

Proof of polynomial bound

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Fix G edge-covered by shortest paths $P_1, \ldots, P_k$.

Choose a path, say P_1.

A path P is parallel to P_1 if there exists a shortest path Q in G containing P as a subpath, whose extremities are on P_1.

Proof of polynomial bound

Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Fix G edge-covered by shortest paths $P_1, \ldots, P_k$.

Choose a path, say P_1.

A path P is parallel to P_1 if there exists a shortest path Q in G containing P as a subpath, whose extremities are on P_1.

Proof of polynomial bound

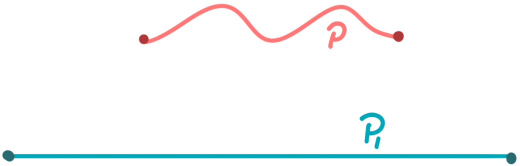
Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Fix G edge-covered by shortest paths $P_1, \ldots, P_k$.

Choose a path, say P_1.

A path P is parallel to P_1 if there exists a shortest path Q in G containing P as a subpath, whose extremities are on P_1.



Proof of polynomial bound

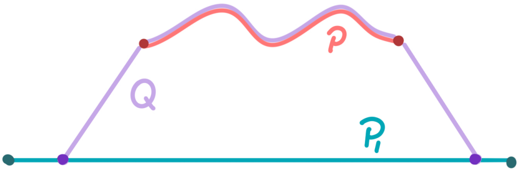
Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Fix G edge-covered by shortest paths $P_1, \ldots, P_k$.

Choose a path, say P_1.

A path P is parallel to P_1 if there exists a shortest path Q in G containing P as a subpath, whose extremities are on P_1.



Proof of polynomial bound

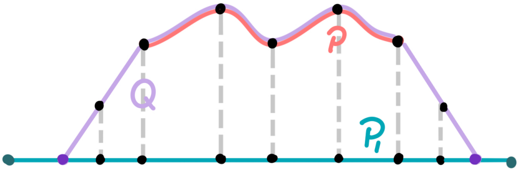
Theorem (Baste, DM, Giocanti, Objois, Picavet 2025)

Let G be a graph edge-coverable by k shortest paths. Then $\mathrm{pw}(G) = O(k^4)$.

Fix G edge-covered by shortest paths $P_1, \ldots, P_k$.

Choose a path, say P_1.

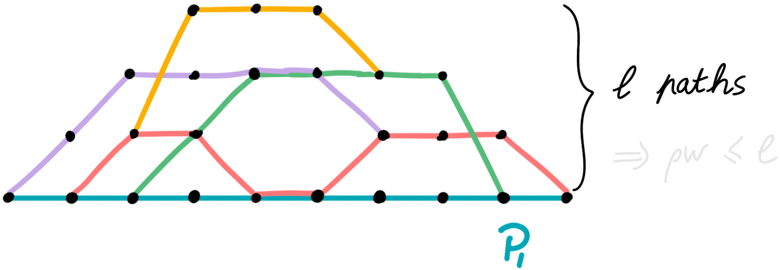
A path P is parallel to P_1 if there exists a shortest path Q in G containing P as a subpath, whose extremities are on P_1.



♡ Parallel paths ♡

Lemma

If G is edge-coverable by ℓ paths all parallel to P_1, then $\mathrm{pw}(G) \leq \ell$.



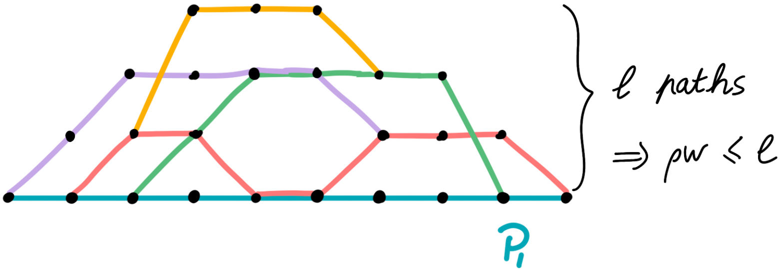
Goal

Find a way to make such structures (graphs coverable by parallel paths) appears.

♡ Parallel paths ♡

Lemma

If G is edge-coverable by ℓ paths all parallel to P_1, then $\mathrm{pw}(G) \leq \ell$.



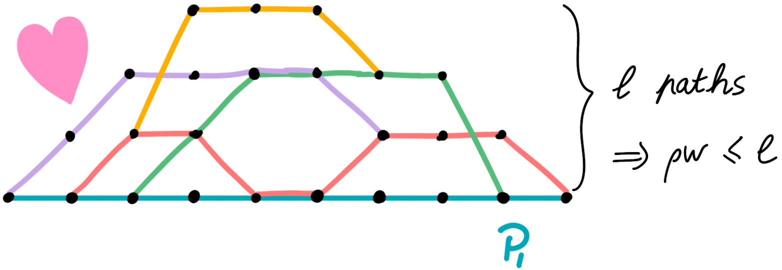
Goal

Find a way to make such structures (graphs coverable by parallel paths) appears.

♡ Parallel paths ♡

Lemma

If G is *edge-coverable by ℓ paths all parallel to P_1, then* $\mathrm{pw}(G) \leq \ell$.



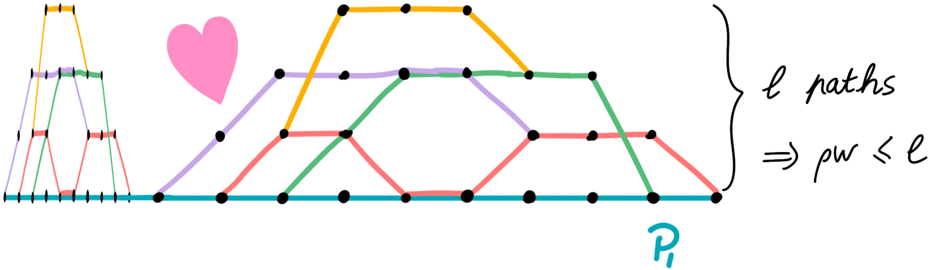
Goal

Find a way to make such structures (graphs coverable by parallel paths) appears.

♡ Parallel paths ♡

Lemma

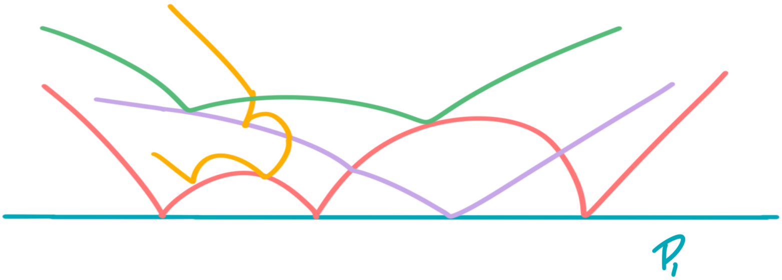
If G is edge-coverable by ℓ paths all parallel to P_1, then $\mathrm{pw}(G) \leq \ell$.



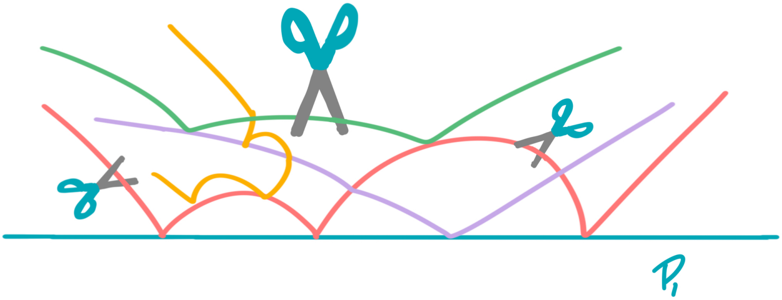
Goal

Find a way to make such structures (graphs coverable by parallel paths) appears.

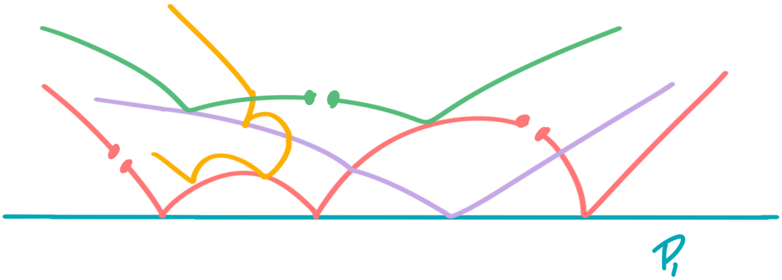
From key lemma. . .



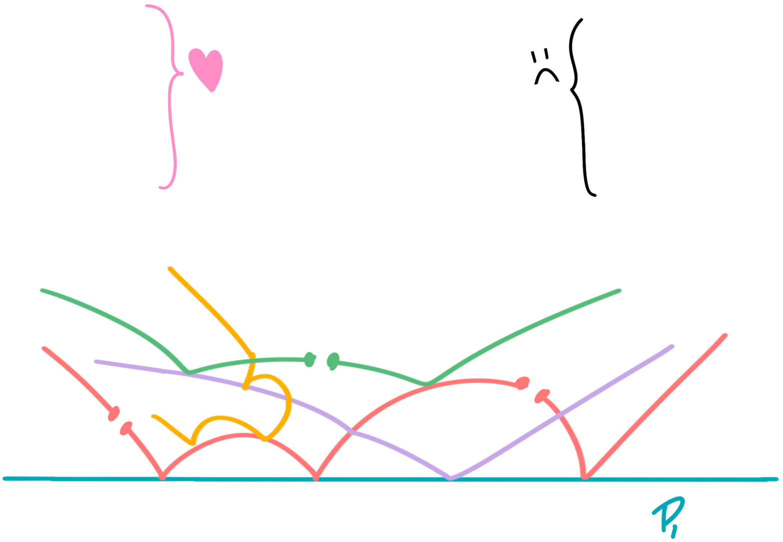
From key lemma. . .



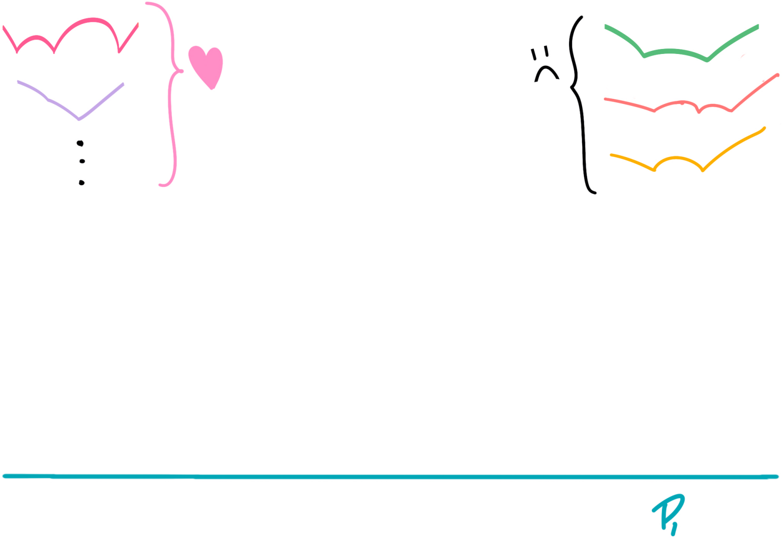
From key lemma. . .



From key lemma. . .

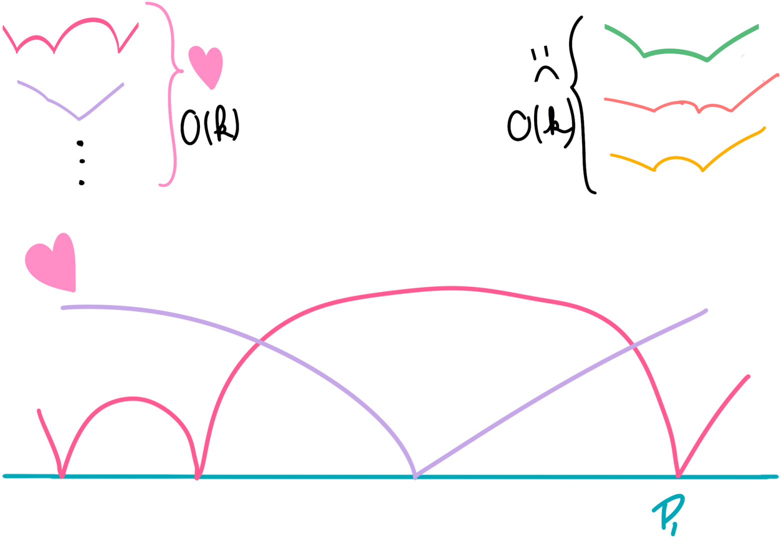


From key lemma...

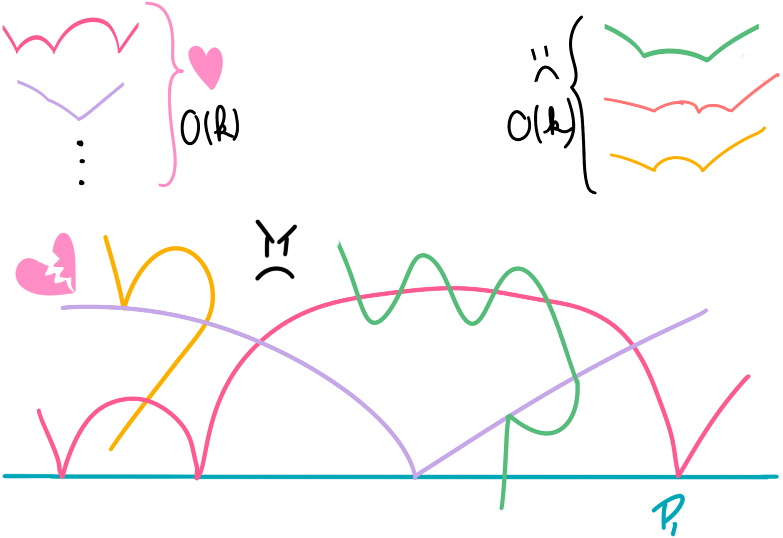


From key lemma. . .

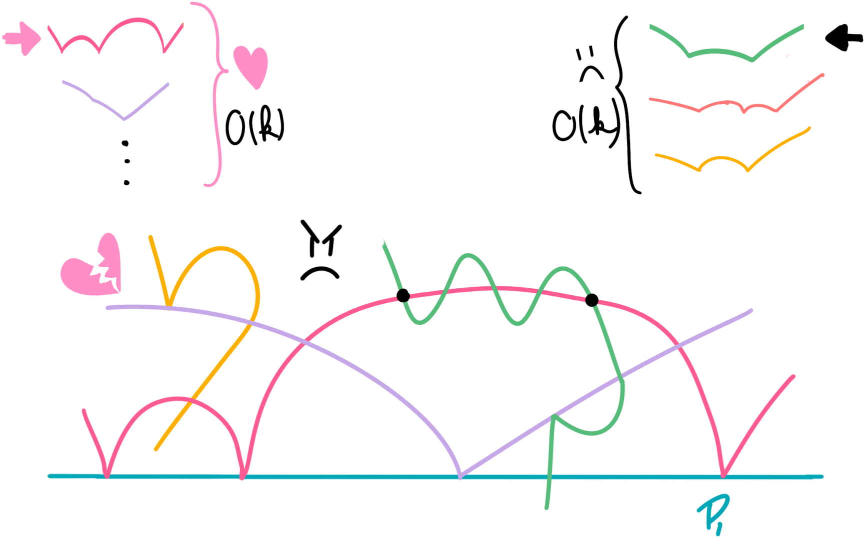
From key lemma. . .



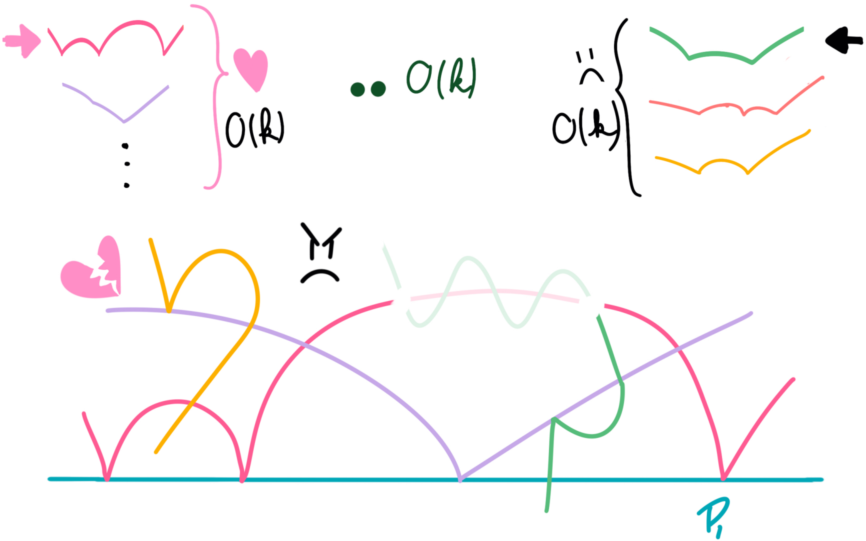
From key lemma. . .



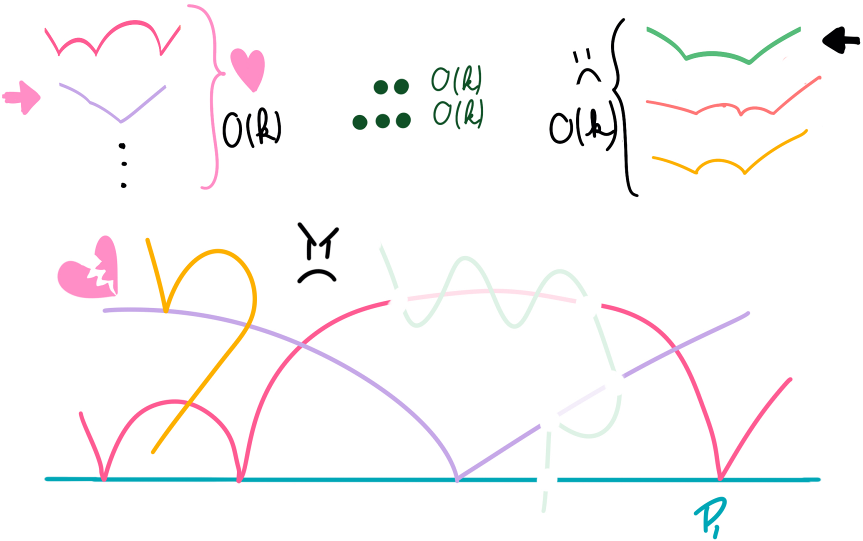
From key lemma. . .



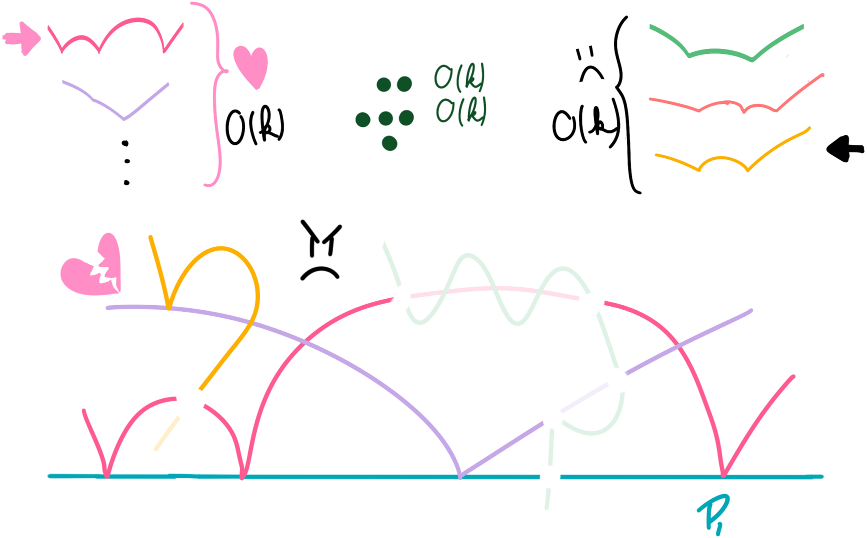
From key lemma. . .



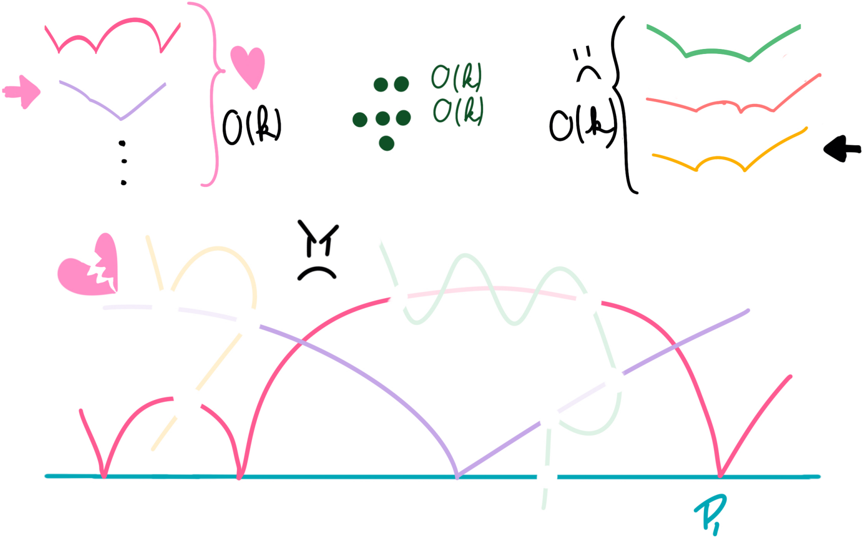
From key lemma. . .



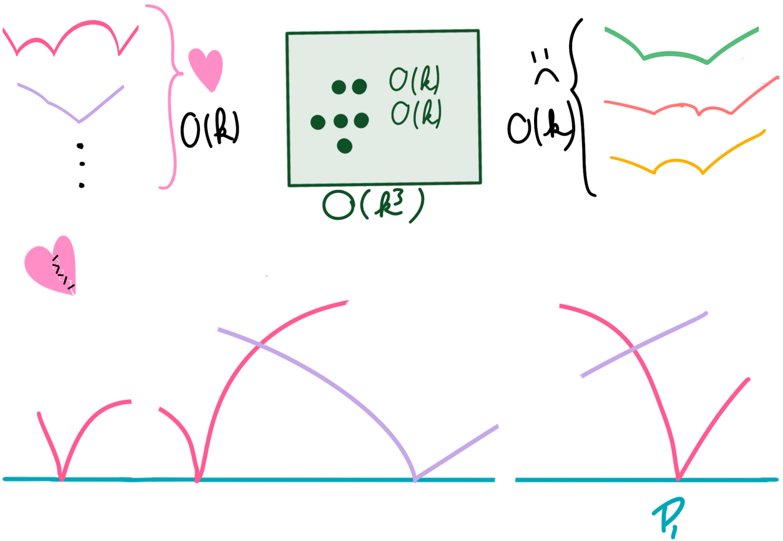
From key lemma. . .



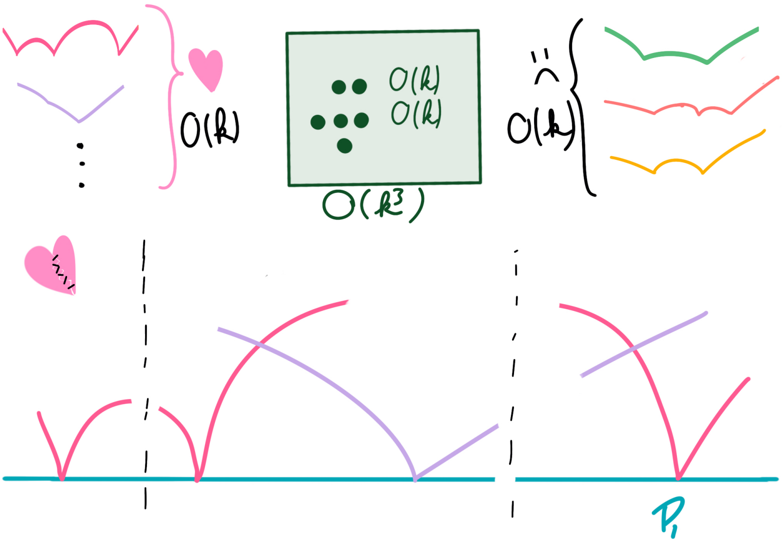
From key lemma. . .



From key lemma...



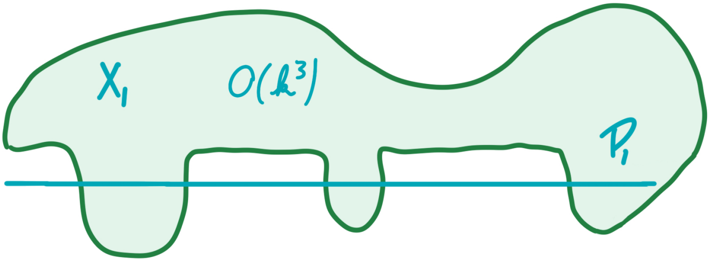
From key lemma...



...to main lemma...

Lemma (Main)

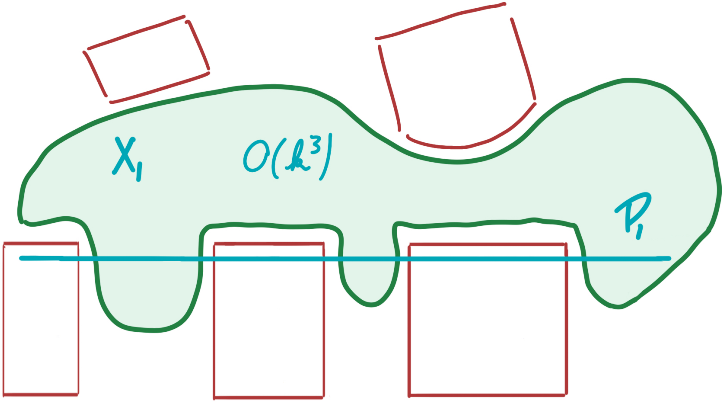
There exists X_1 *of size* $O(k^3)$ *such that every component of* $G - X_1$ *intersecting* P_1 *consists of* $O(k)$ *paths all parallel to* P_1.



. . . to main lemma. . .

Lemma (Main)

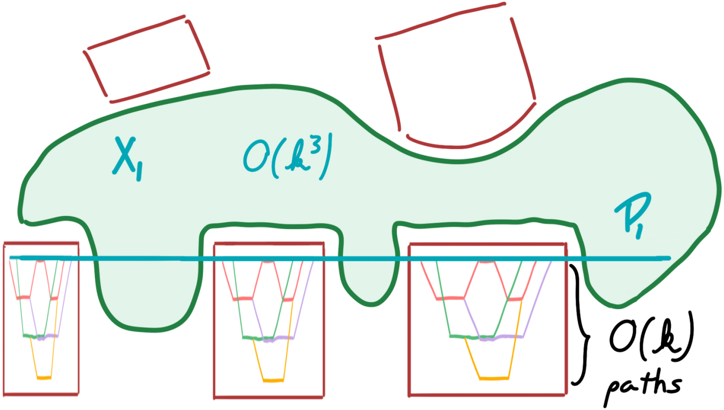
There exists X_1 of size $O(k^3)$ such that every component of $G - X_1$ intersecting P_1 consists of $O(k)$ paths all parallel to P_1.



. . . to main lemma. . .

Lemma (Main)

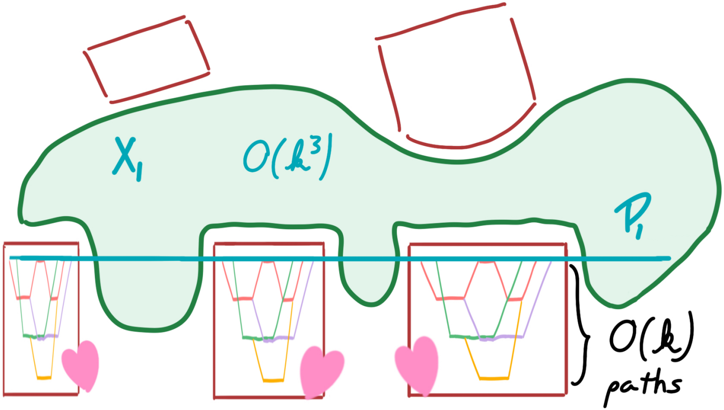
There exists X_1 of size $O(k^3)$ such that every component of $G - X_1$ intersecting P_1 consists of $O(k)$ paths all parallel to P_1.



. . . to main lemma. . .

Lemma (Main)

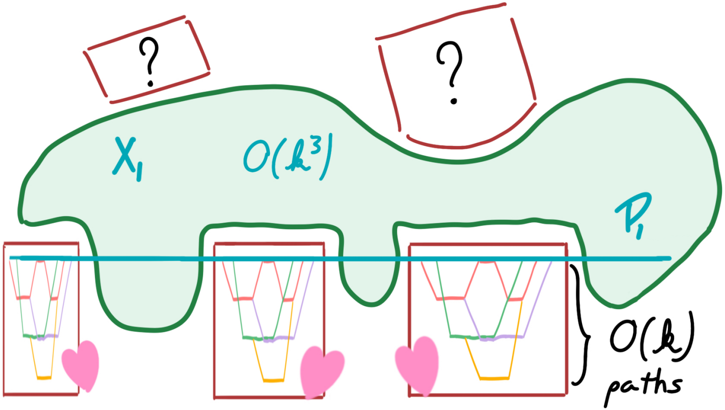
There exists X_1 of size $O(k^3)$ such that every component of $G - X_1$ intersecting P_1 consists of $O(k)$ paths all parallel to P_1.



. . . to main lemma. . .

Lemma (Main)

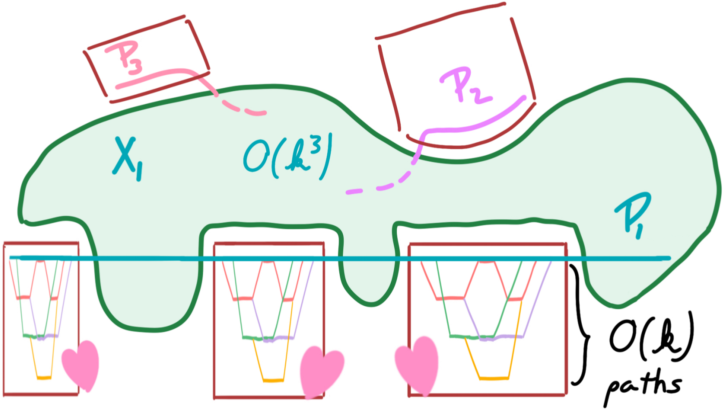
There exists X_1 of size $O(k^3)$ such that every component of $G - X_1$ intersecting P_1 consists of $O(k)$ paths all parallel to P_1.



. . . to main lemma. . .

Lemma (Main)

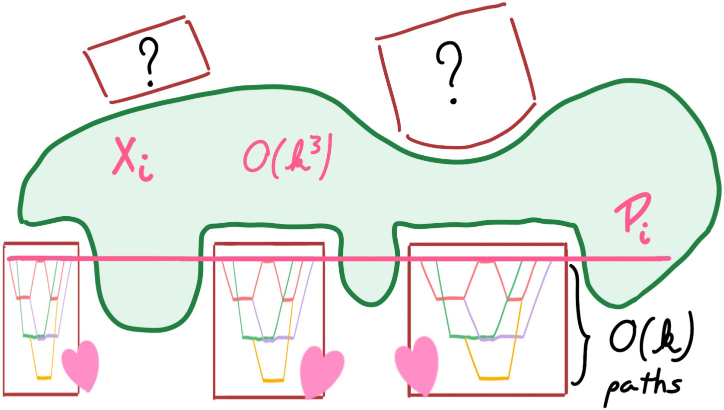
There exists X_1 of size $O(k^3)$ such that every component of $G - X_1$ intersecting P_1 consists of $O(k)$ paths all parallel to P_1.



. . . to main lemma. . .

Lemma (Main)

There exists X_i of size $O(k^3)$ such that every component of $G - X_i$ intersecting P_i consists of $O(k)$ paths all parallel to P_i, for $i = 1, \ldots, k$.



. . . to main theorem

Lemma

There exists $X := \bigcup_{i=1}^{k} X_i$ *of size* $k \cdot O(k^3)$ *such that every component of* $G - X$ *consists of* $O(k)$ *paths all parallel to some* P_i.

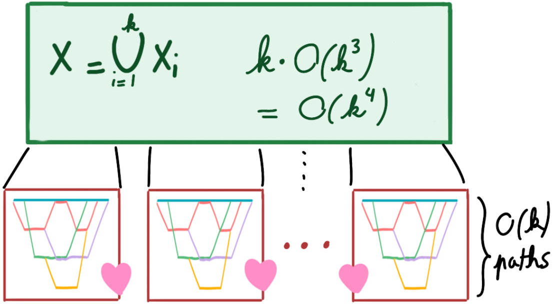
$$X = \bigcup_{i=1}^{k} X_i \qquad k \cdot O(k^3) = O(k^4)$$

A graph edge-coverable by k shortest path has pathwidth $O(k^4)$.

... to main theorem

Lemma

There exists $X := \bigcup_{i=1}^{k} X_i$ *of size* $k \cdot O(k^3)$ *such that every component of* $G - X$ *consists of* $O(k)$ *paths all parallel to some* P_i.

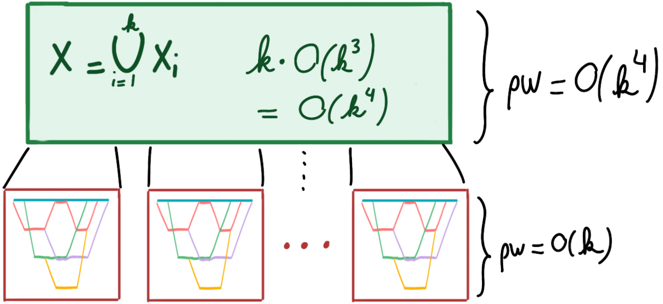


A graph edge-coverable by k shortest path has pathwidth $O(k^4)$.

...to main theorem

Lemma

There exists $X := \bigcup_{i=1}^{k} X_i$ *of size* $k \cdot O(k^3)$ *such that every component of* $G - X$ *consists of* $O(k)$ *paths all parallel to some* P_i.

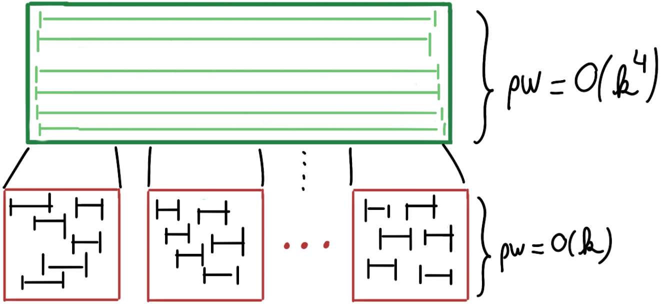


A graph edge-coverable by k shortest path has pathwidth $O(k^4)$.

. . . to main theorem

Lemma

There exists $X := \bigcup_{i=1}^{k} X_i$ *of size* $k \cdot O(k^3)$ *such that every component of* $G - X$ *consists of* $O(k)$ *paths all parallel to some* P_i.

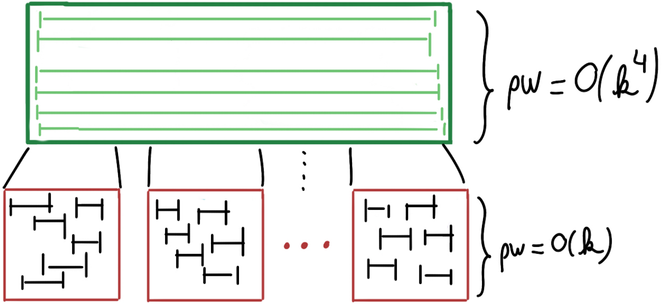


A graph edge-coverable by k shortest path has pathwidth $O(k^4)$.

. . . to main theorem

Lemma

There exists $X := \bigcup_{i=1}^{k} X_i$ *of size* $k \cdot O(k^3)$ *such that every component of* $G - X$ *consists of* $O(k)$ *paths all parallel to some* P_i.



A graph edge-coverable by k shortest path has pathwidth $O(k^4)$.

Isometric path vertex-cover

Covering the vertices with shortest paths?

Isometric path partition

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ vertex-disjoint shortest paths?

Isometric path cover

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ shortest paths?

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph vertex-coverable by k shortest paths. Then, $\text{pw}(G) = O(k \cdot 3^k)$.

Isometric path vertex-cover

Covering the vertices with shortest paths?

Isometric path partition

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ vertex-disjoint shortest paths?

Isometric path cover

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ shortest paths?

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph vertex-coverable by k shortest paths. Then, $\mathrm{pw}(G) = O(k \cdot 3^k)$.

Isometric path vertex-cover

Covering the vertices with shortest paths?

Isometric path partition

`In:` Graph G, integer k.

`Out:` Can the vertices of G be covered by $\leq k$ vertex-disjoint shortest paths?

Isometric path cover

`In:` Graph G, integer k.

`Out:` Can the vertices of G be covered by $\leq k$ shortest paths?

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph vertex-coverable by k shortest paths. Then, $\mathrm{pw}(G) = O(k \cdot 3^k)$.

Isometric path vertex-cover

Covering the vertices with shortest paths?

Isometric path partition

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ vertex-disjoint shortest paths?

Isometric path cover

In: Graph G, integer k.

Out: Can the vertices of G be covered by $\leq k$ shortest paths?

Theorem (Dumas, Foucaud, Perez, Todinca 2024)

Let G be a graph vertex-coverable by k shortest paths. Then, $\mathrm{pw}(G) = O(k \cdot 3^k)$.

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\text{pw} \leq O(k \cdot 3^k)$

Isometric path cover:
NP-complete
XP param. by k

Isometric path partition:
NP-complete
W[1]-hard for pw
XP: $n^{O(\text{tw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\text{pw} \leq O(k^4)$

Isometric path edge-cover:
(Parametrized) complexity?

Isometric path edge-partition:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\text{pw} \leq O(k \cdot 3^k)$

ISOMETRIC PATH COVER:
NP-complete
XP param. by k

ISOMETRIC PATH PARTITION:
NP-complete
W[1]-hard for pw
XP: $n^{O(\text{tw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\text{pw} \leq O(k^4)$

ISOMETRIC PATH EDGE-COVER:
(Parametrized) complexity?

ISOMETRIC PATH EDGE-PARTITION:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$$\text{pw} \leq O(k \cdot 3^k)$$

ISOMETRIC PATH COVER:
NP-complete
XP param. by k

ISOMETRIC PATH PARTITION:
NP-complete
W[1]-hard for pw
XP: $n^{O(\text{tw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$$\text{pw} \leq O(k^4)$$

ISOMETRIC PATH EDGE-COVER:
(Parametrized) complexity?

ISOMETRIC PATH EDGE-PARTITION:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\mathrm{pw} \leq O(k \cdot 3^k)$

Isometric path cover:
NP-complete
XP param. by k

Isometric path partition:
NP-complete
W[1]-hard for pw
XP: $n^{O(\mathrm{tw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\mathrm{pw} \leq O(k^4)$

Isometric path edge-cover:
(Parametrized) complexity?

Isometric path edge-partition:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\text{pw} \leq O(k \cdot 3^k)$

Isometric path cover:
NP-complete
XP param. by k

Isometric path partition:
NP-complete
W[1]-hard for pw
XP: $n^{O(\text{pw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\text{pw} \leq O(k^4)$

Isometric path edge-cover:
(Parametrized) complexity?

Isometric path edge-partition:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\mathrm{pw} \leq O(k \cdot 3^k)$

ISOMETRIC PATH COVER:
NP-complete
XP param. by k

ISOMETRIC PATH PARTITION:
NP-complete
W[1]-hard for pw
XP: $n^{O(\mathrm{pw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\mathrm{pw} \leq O(k^4)$

ISOMETRIC PATH EDGE-COVER:
(Parametrized) complexity?

ISOMETRIC PATH EDGE-PARTITION:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$\mathrm{pw} \leq O(k \cdot 3^k)$

ISOMETRIC PATH COVER:
NP-complete
XP param. by k

ISOMETRIC PATH PARTITION:
NP-complete
W[1]-hard for pw
XP: $n^{O(\mathrm{pw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$\mathrm{pw} \leq O(k^4)$

ISOMETRIC PATH EDGE-COVER:
(Parametrized) complexity?

ISOMETRIC PATH EDGE-PARTITION:
(Parametrized) complexity?

Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Vertex vs. Edge

Path vertex-cover

Vertex-coverable by k shortest paths:

$$\text{pw} \leq O(k \cdot 3^k)$$

Isometric path cover:
NP-complete
XP param. by k

Isometric path partition:
NP-complete
W[1]-hard for pw
XP: $n^{O(\text{pw}(G))}$

Path edge-cover

Edge-coverable by k shortest paths:

$$\text{pw} \leq O(k^4)$$

Isometric path edge-cover:
(Parametrized) complexity?

Isometric path edge-partition:
(Parametrized) complexity?

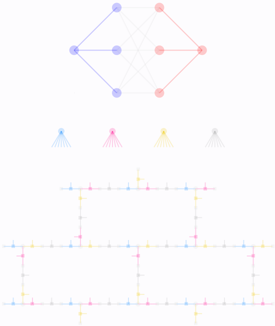
Graphs vertex/edge-coverable by k shortest paths and pathwidth k

Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\implies$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\implies$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?

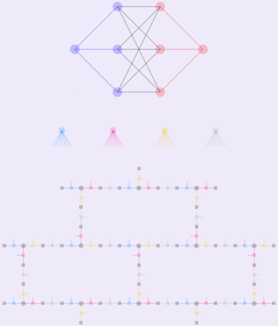


Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\implies$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\implies$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?

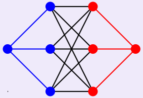


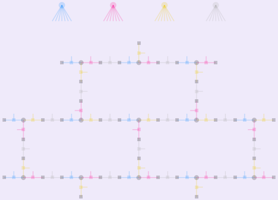
Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\implies$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\implies$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?



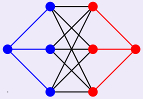


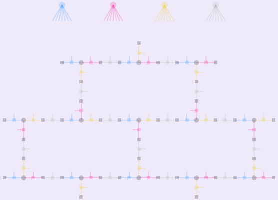
Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\Longrightarrow$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\Longrightarrow$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?



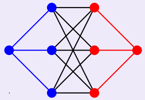


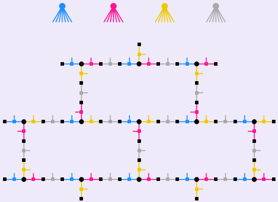
Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\implies$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\implies$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?



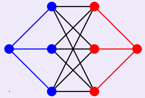


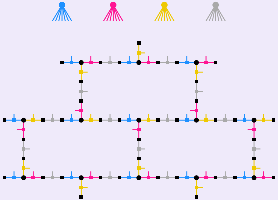
Path to trees?

Shortest paths	Isometric trees (all shortest paths in the tree are shortest paths in G)
Pathwidth	Treewidth (measures distance to a tree)

Treewidth of graphs vertex/edge-coverable by k isometric subtrees?

- unbounded treewidth for vertex-coverability,
- edge-coverable by 2 isometric trees $\implies$ treewidth ≤ 2 (BDGOP),
- edge-coverable by ≥ 4 isometric trees $\implies$ unbounded treewidth (Bastide et al. 2025),
- $k = 3$?





Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

Conclusion

- A graph edge-coverable by k shortest paths has pathwidth $O(k^4)$.
- A graph edge-coverable by 3 shortest paths has pathwidth ≤ 3.
- A graph edge-coverable by 2 isometric trees paths has treewidth ≤ 2.

- Polynomial bounds on pathwidth for graphs vertex-coverable by k shortest paths?
- Linear bounds? (NOT less since there exist graphs edge-coverable by k shortest paths, and pathwidth k).
- Treewidth of graphs edge-coverable by 3 isometric trees?

END

Thanks for your attention