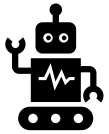
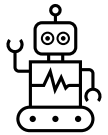
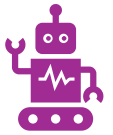
Optimal Deterministic Rendezvous in Labeled Lines

Yann Bourreau, Ananth Narayanan, Alexandre Nolin

The Rendez-Vous Problem







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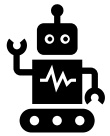

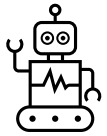

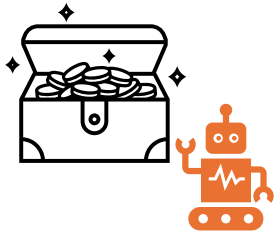

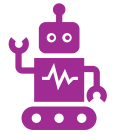

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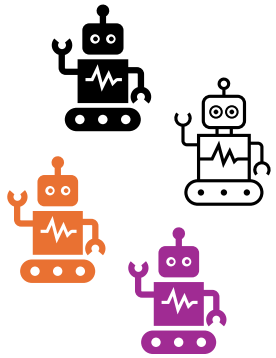

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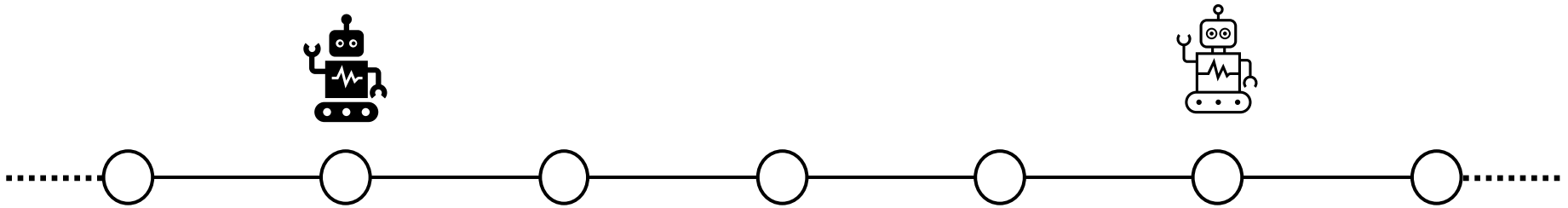

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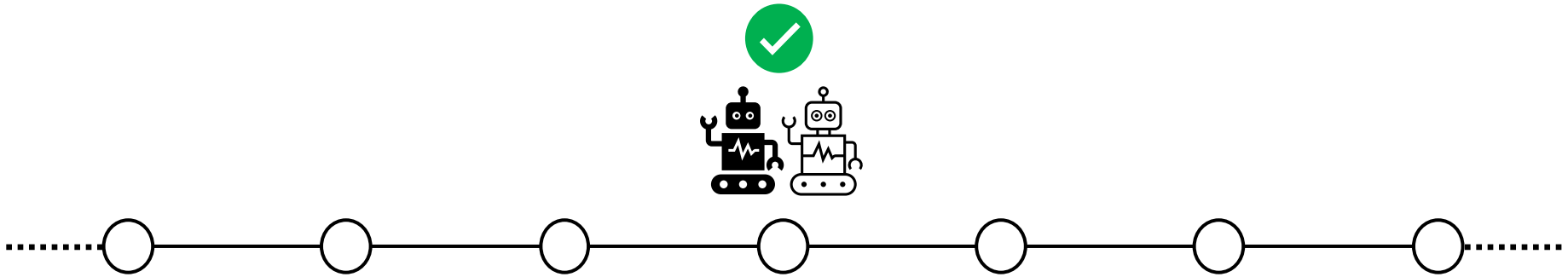
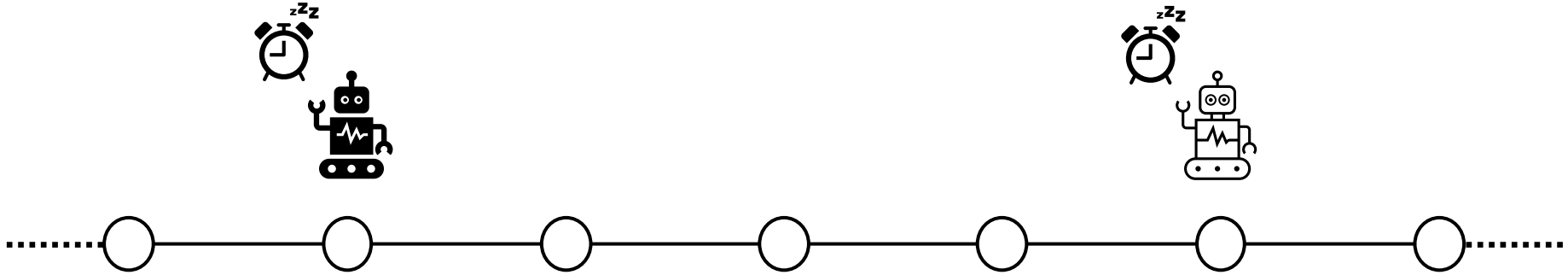
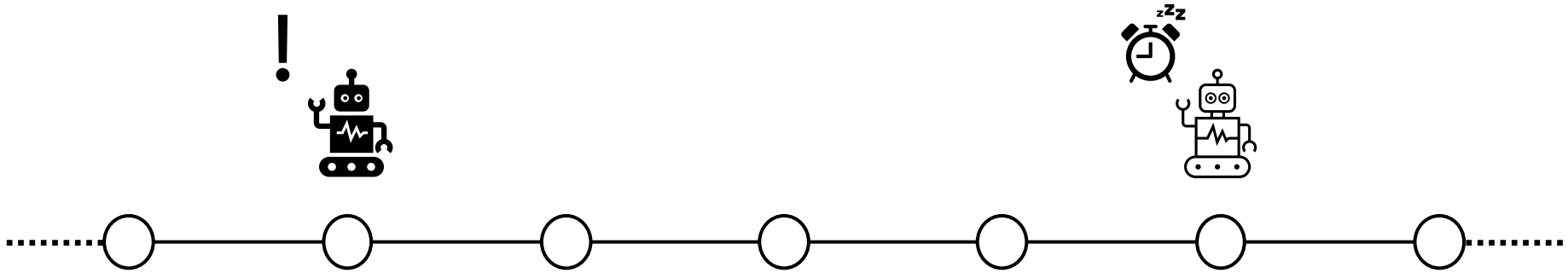


Model settings



- Adversarily chosen wake up time
- Synchronous rounds

Model settings



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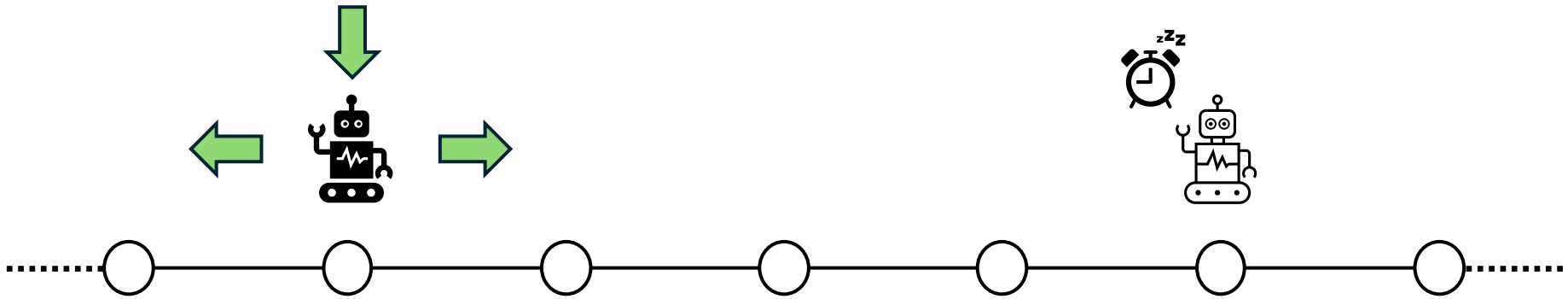

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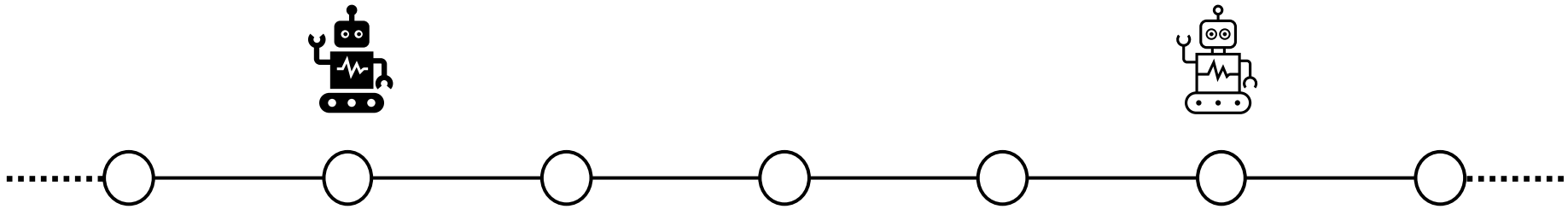

- Adversarily chosen wake up time
- Synchronous rounds
- Rendez-vous: Same node during same round

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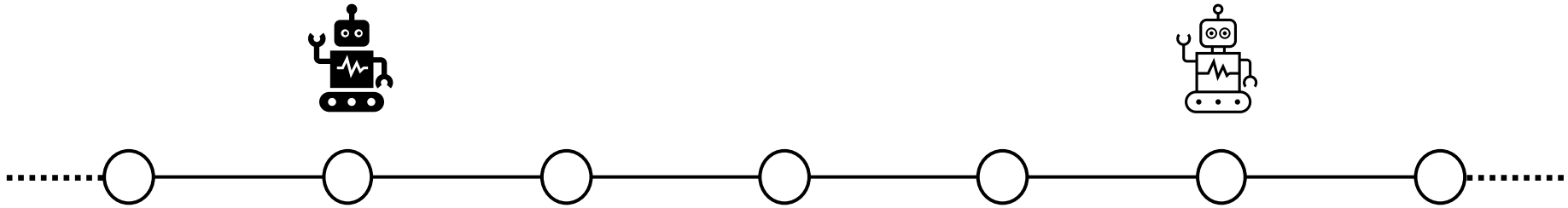
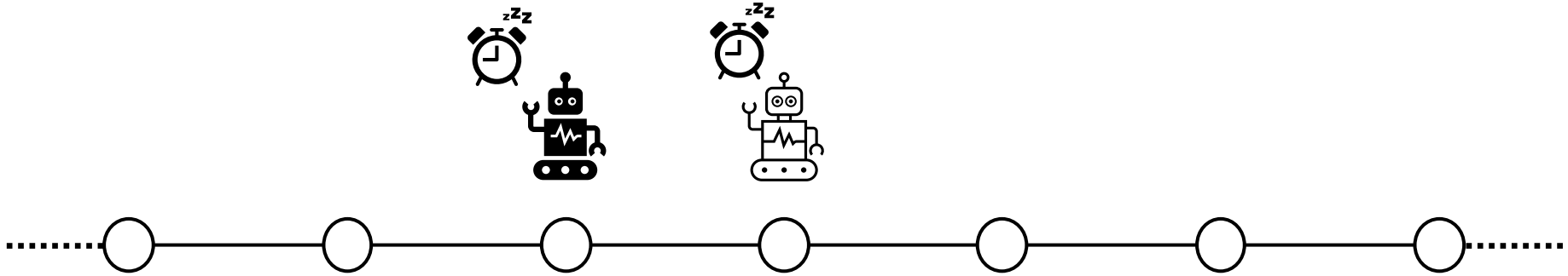


- Adversarily chosen wake up time
- Synchronous rounds
- Rendez-vous: Same node during same round
- Same **deterministic** algorithm
- Robots see each other only on the same node

Why do we need IDs?



- Adversarily chosen wake up time
- Synchronous rounds
- Rendez-vous: Same node during same round
- Same **deterministic** algorithm
- Robots see what is on their node

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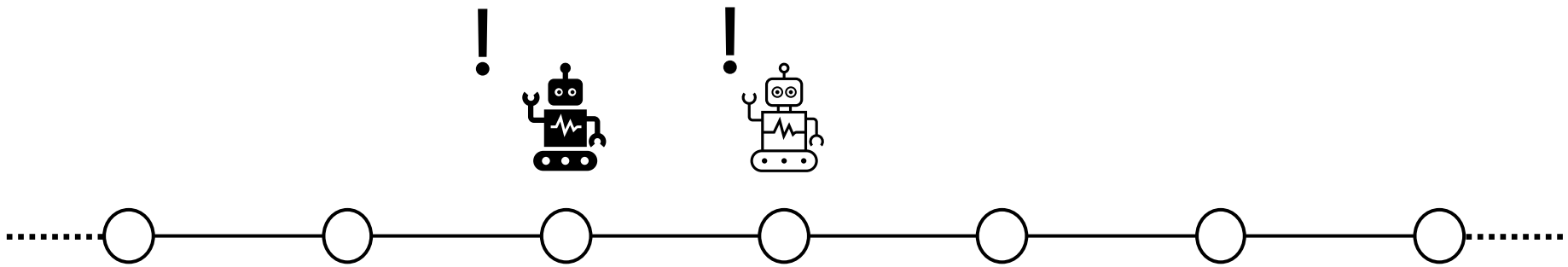

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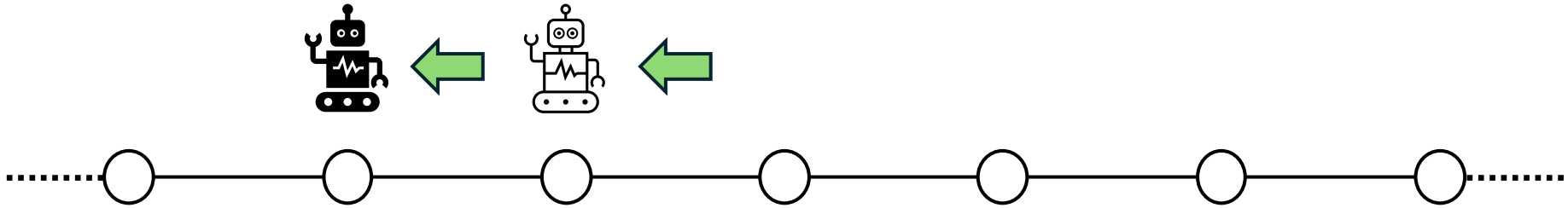

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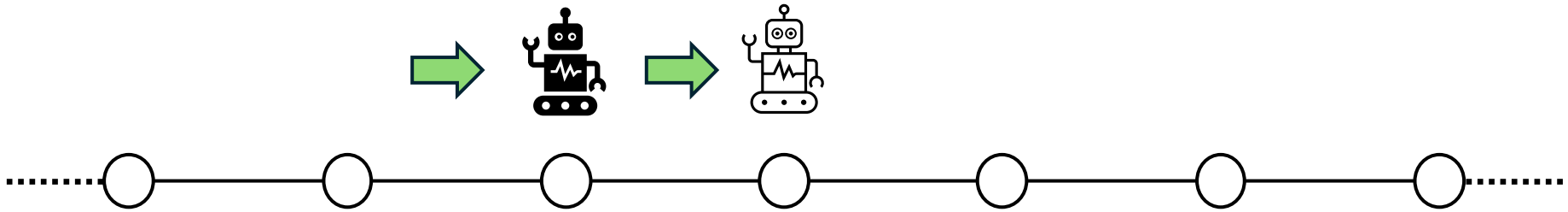

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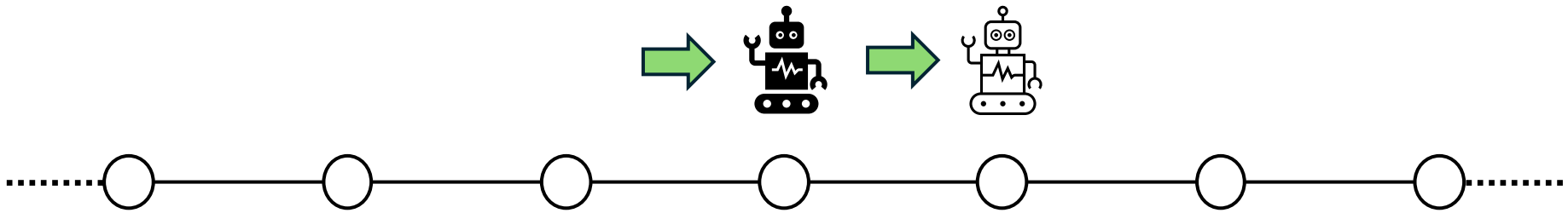
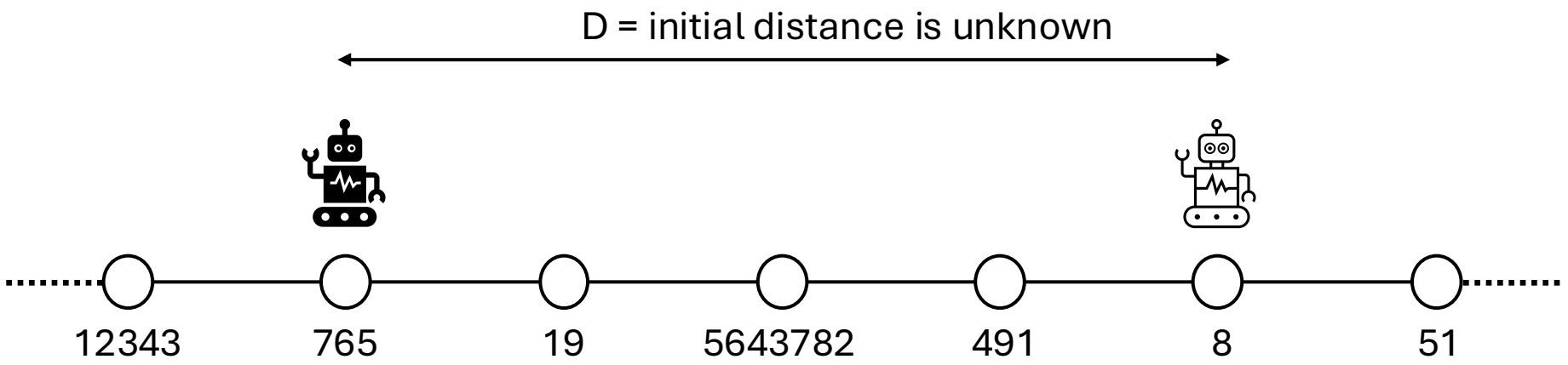


- Adversarily chosen wake up time
- Synchronous rounds
- Rendez-vous: Same node during same round
- Same **deterministic** algorithm
- Robots see what is on their node

not enough to achieve Rendez-vous

Problem setting

Synchronous rounds and infinite memory



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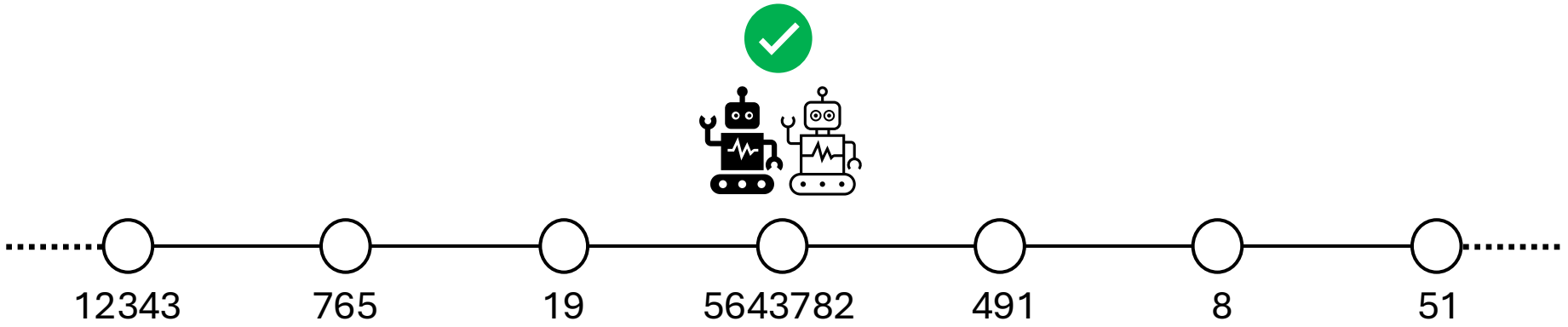

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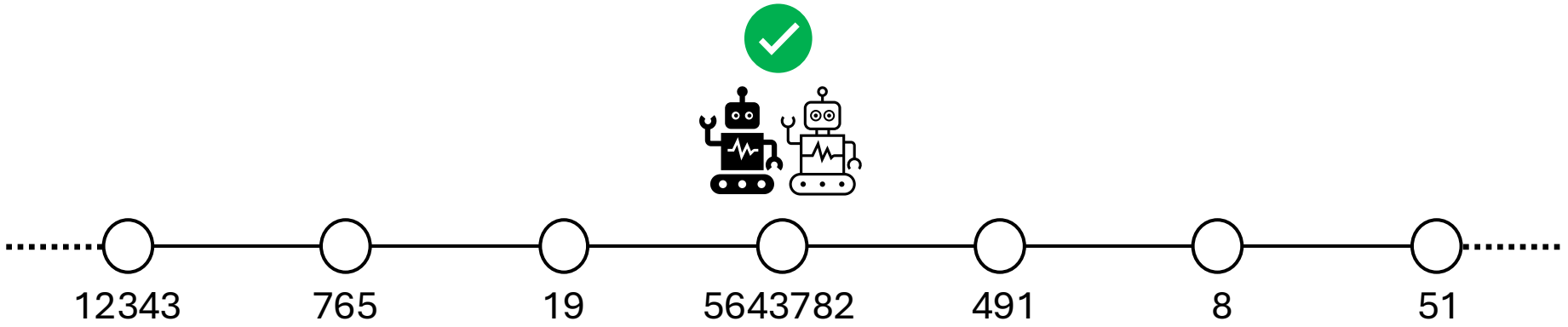
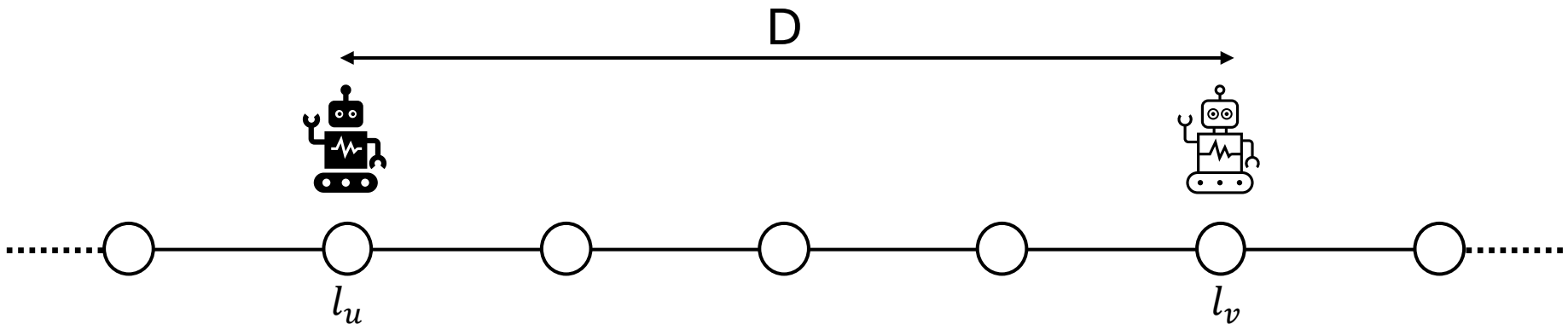


How long does it take?

Previous work and our results

$$l_{max} = max(l_u, l_v)$$



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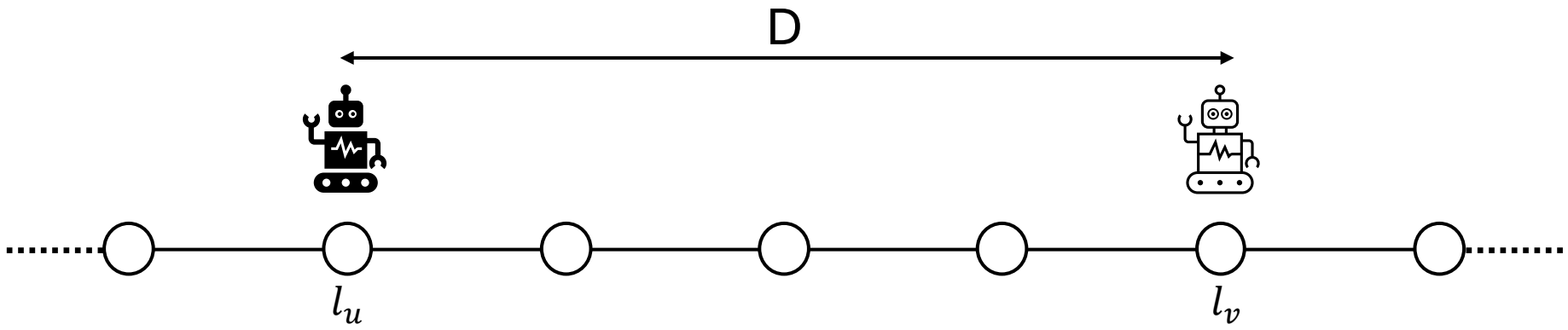
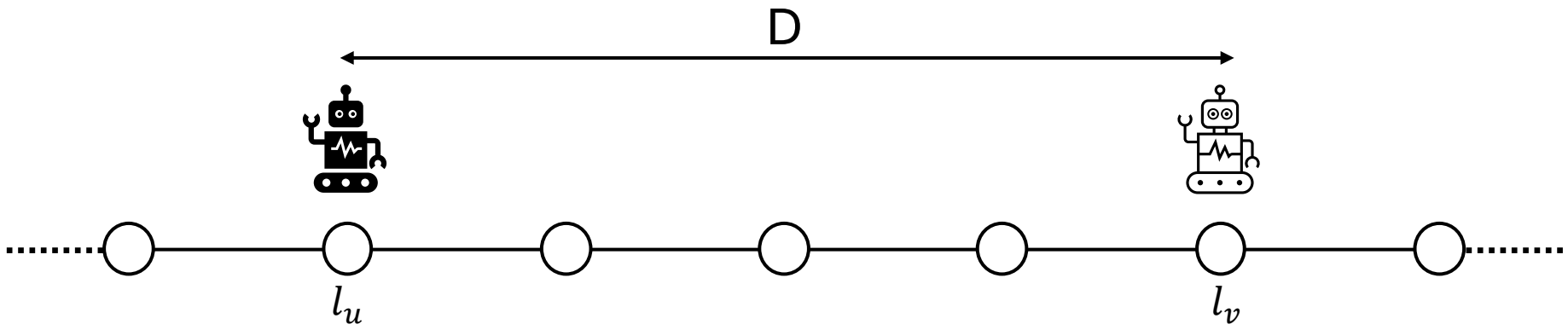


- $\Omega(D \cdot \log^*(l_{max}))$ - Miller and Pelc 2023
- $O(D^2 \cdot (\log^*(l_{max}))^3)$ - Miller and Pelc 2023

Measured when first agent starts

Previous work and our results

$$l_{max} = max(l_u, l_v)$$



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$$\log^* \mathrm{k} = \begin{cases} 0 \text{ if } \mathrm{k} \leq 1 \\ 1 + \log * (\log \mathrm{k}) \text{ if } \mathrm{k} > 1 \end{cases}$$

Previous work and our results

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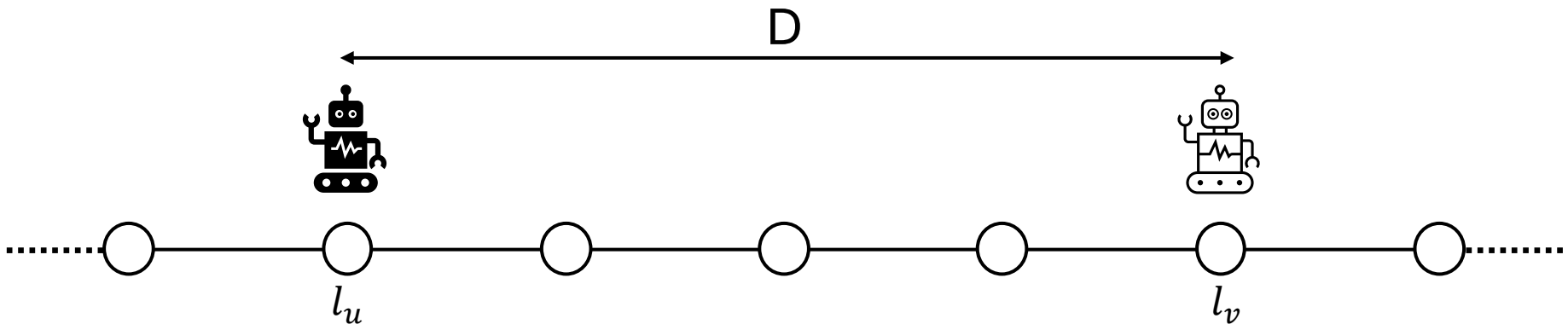

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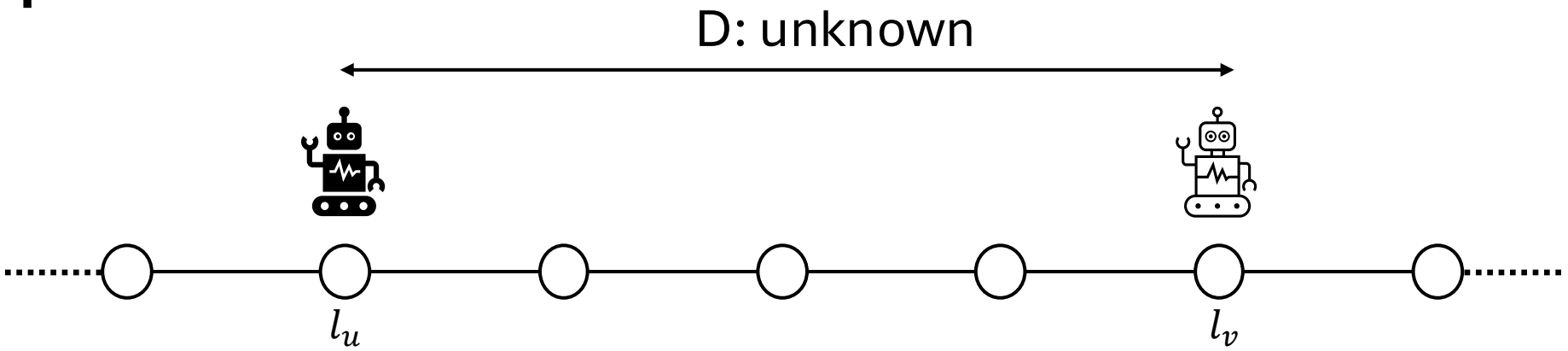
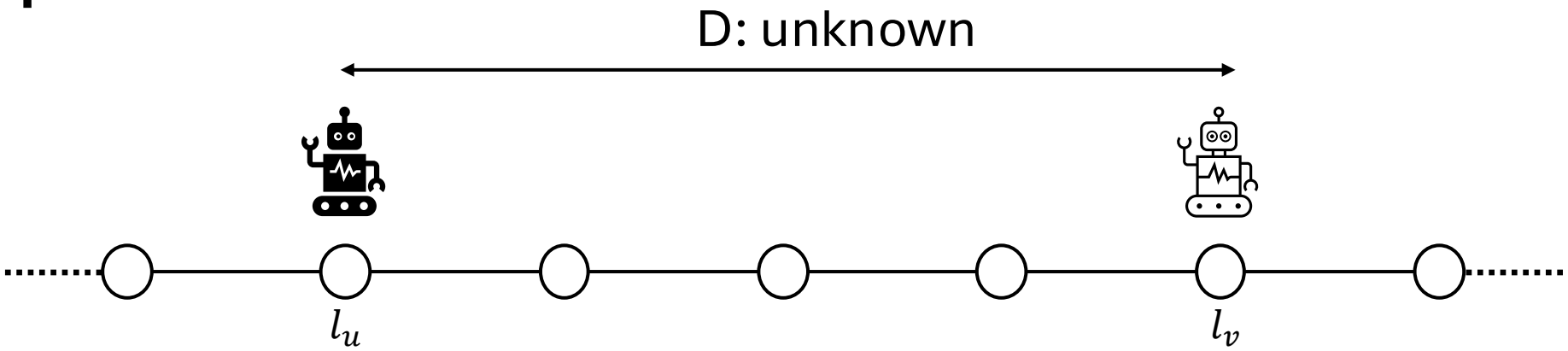
Our result: $O(\mathrm{D} \cdot \log^*(l_{max}))$

Simple case



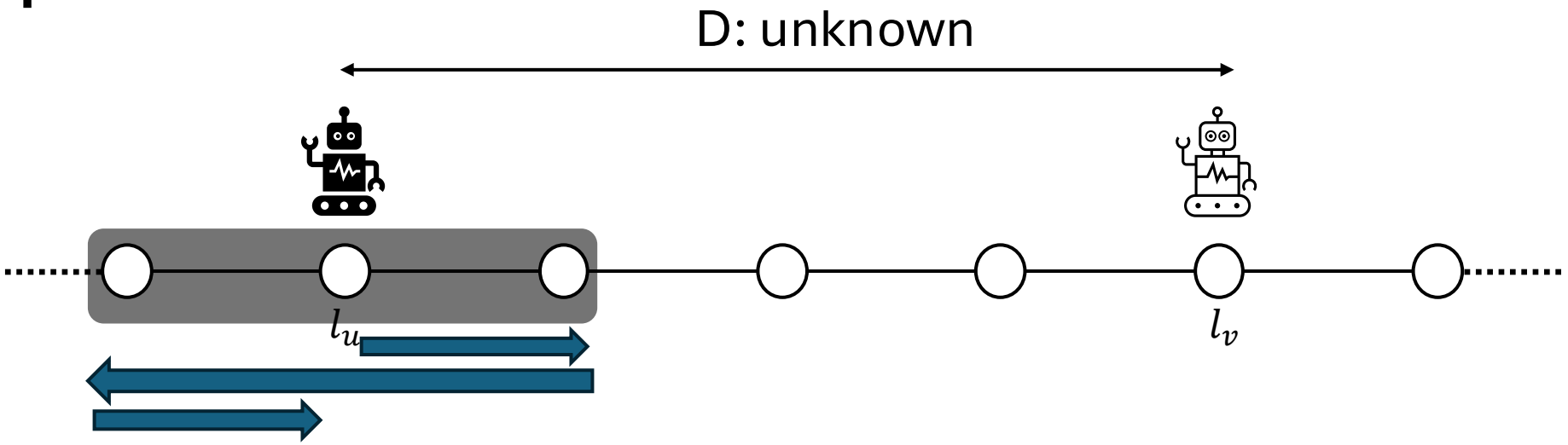
- 2 different algorithms and same wake up time

Simple case



- 2 different algorithms and same wake up time
- Have one robot search and the other one stay!

Simple case



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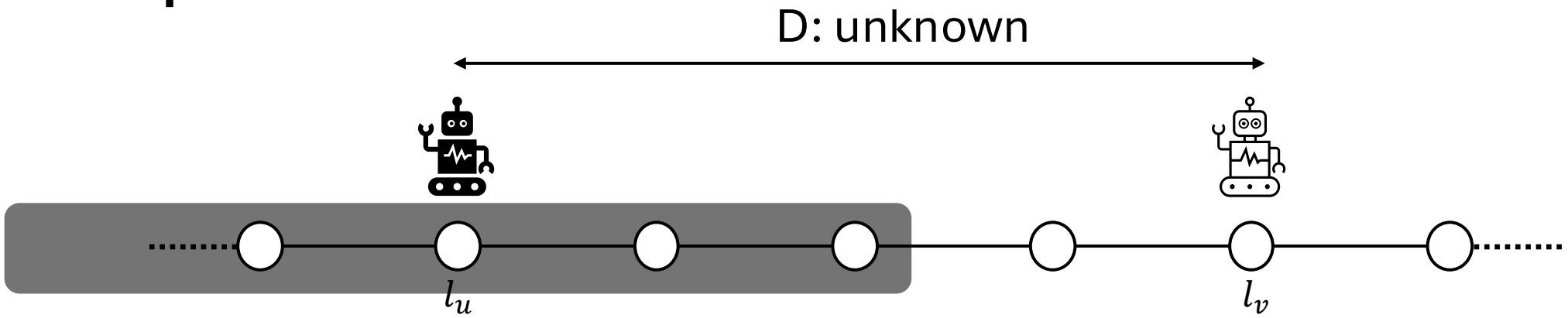

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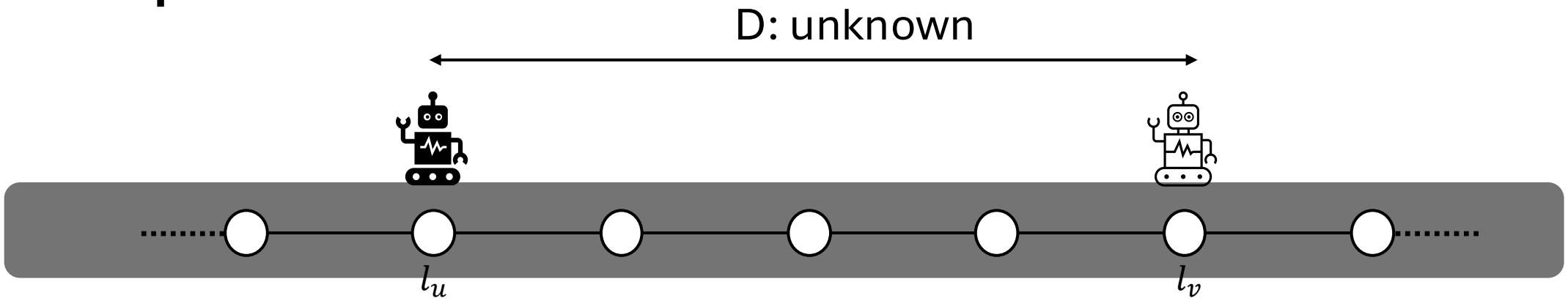

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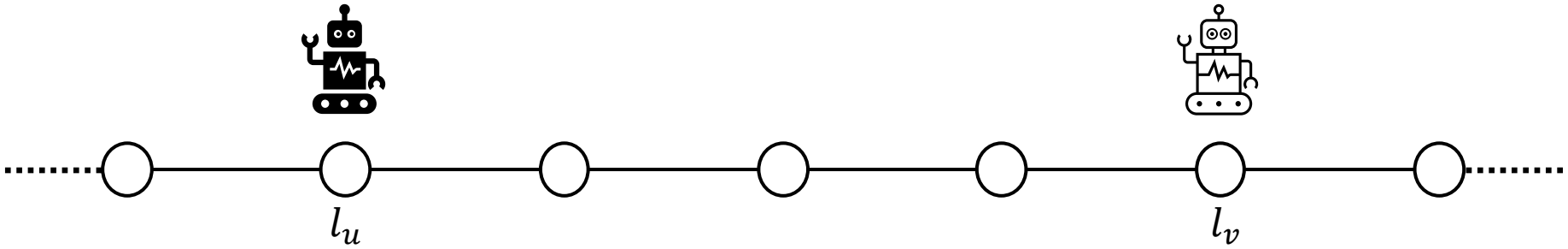
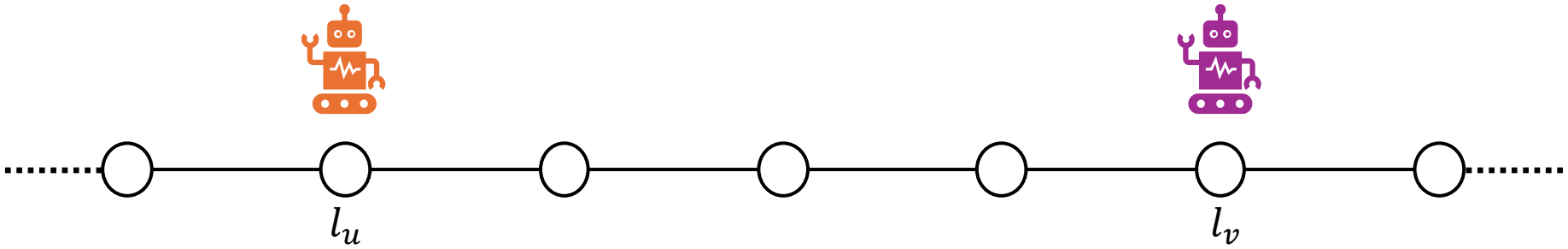
Rendez-Vous: O(D)

Algorithm sketch



1. Color each agent

Algorithm sketch



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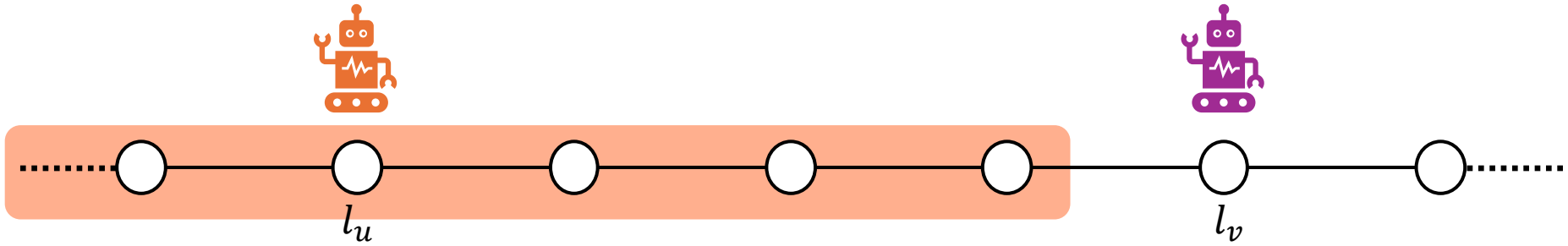

1. Color each agent
2. Iterate through color class -> search k-hop radius

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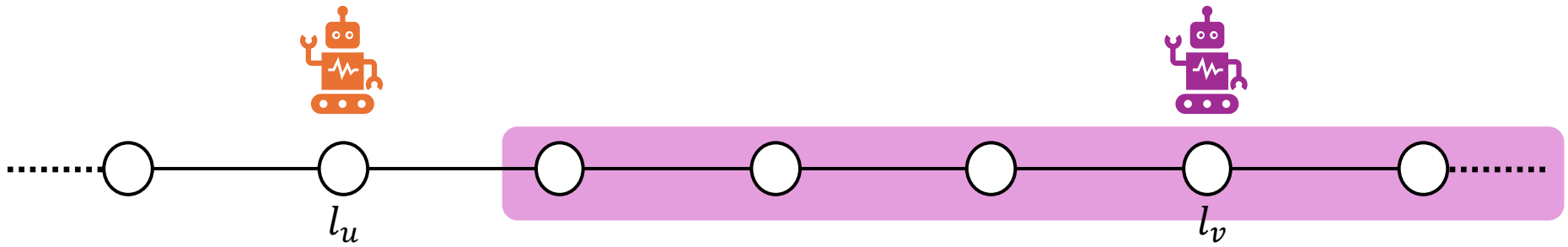

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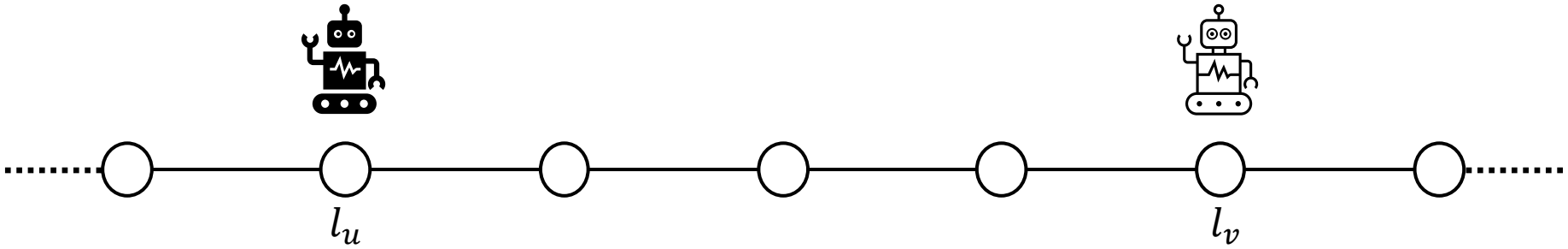

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3. Double k and move to step 1

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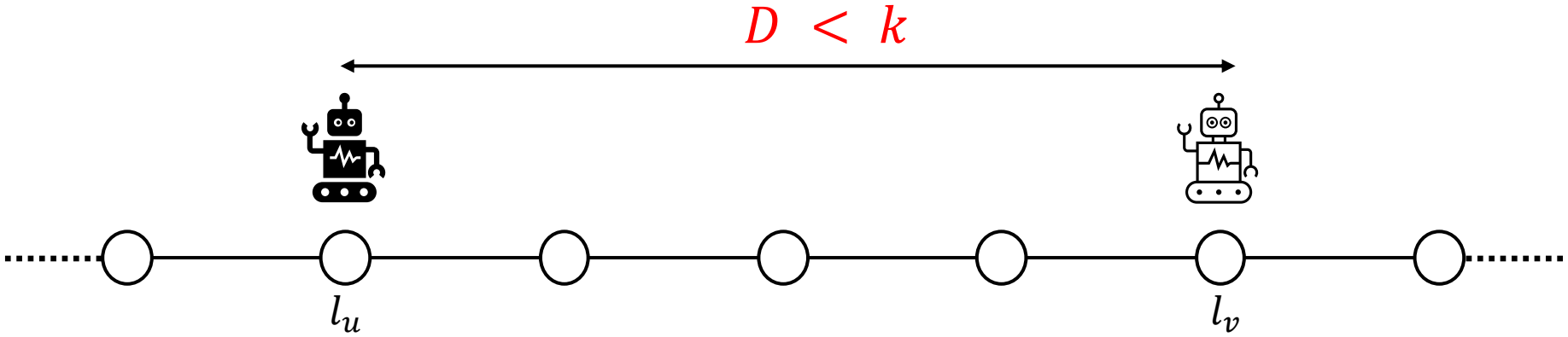

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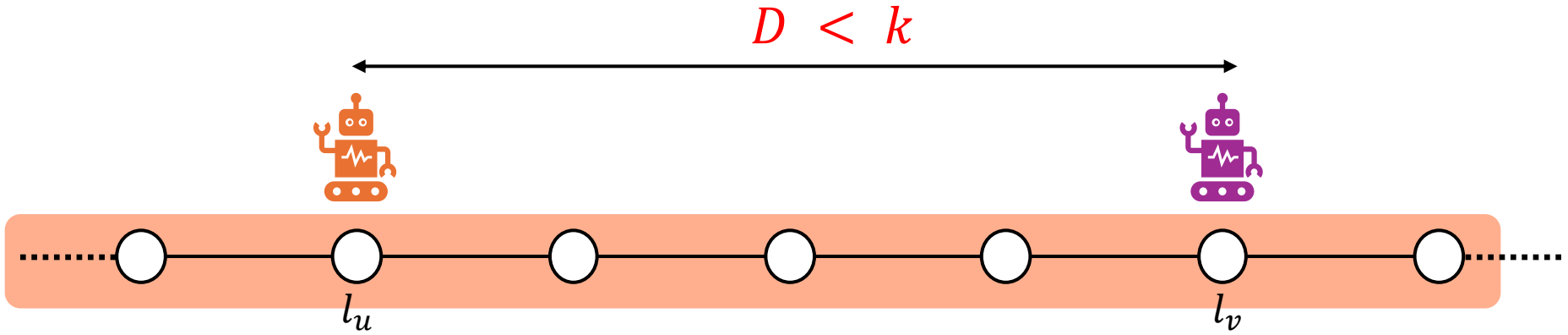

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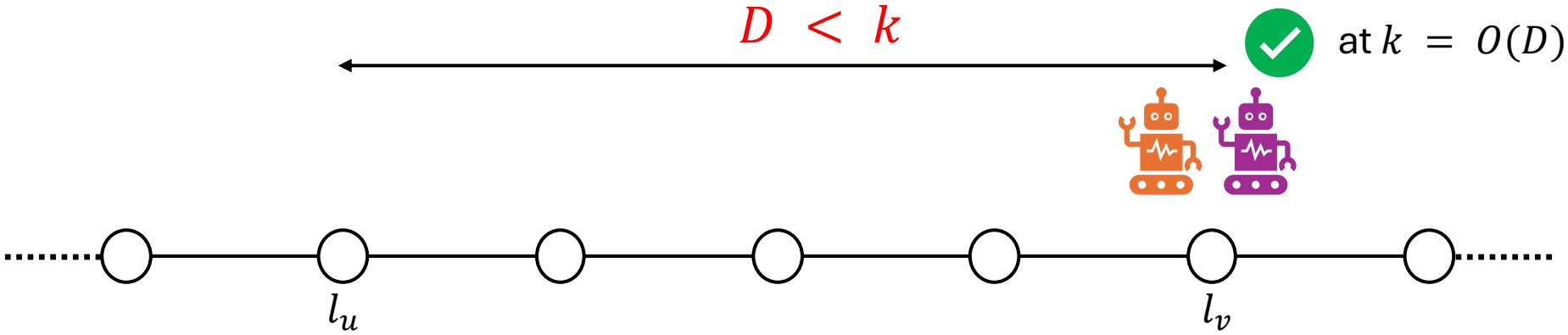

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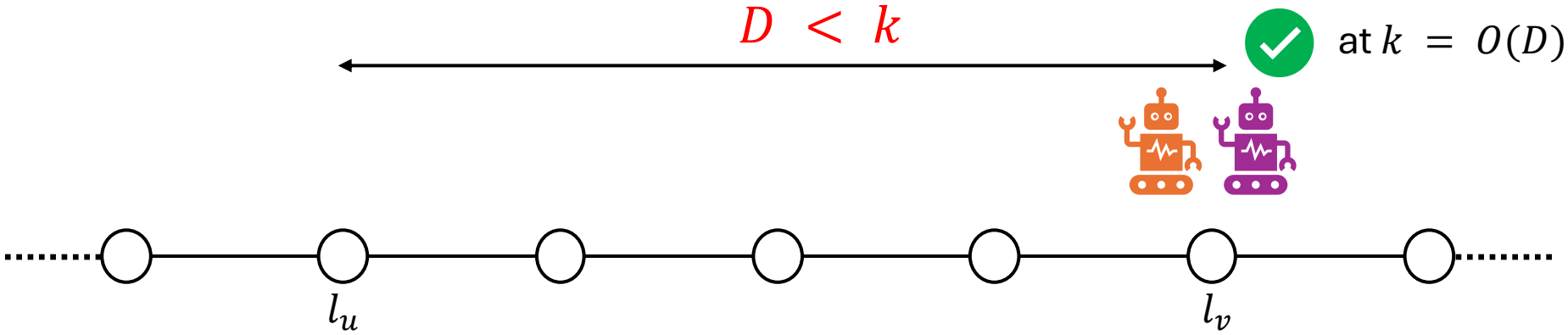

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Runtime: $\underbrace{\textbf{Searching Phase}}_{O(D)}$ **+** **Coloring Phase**

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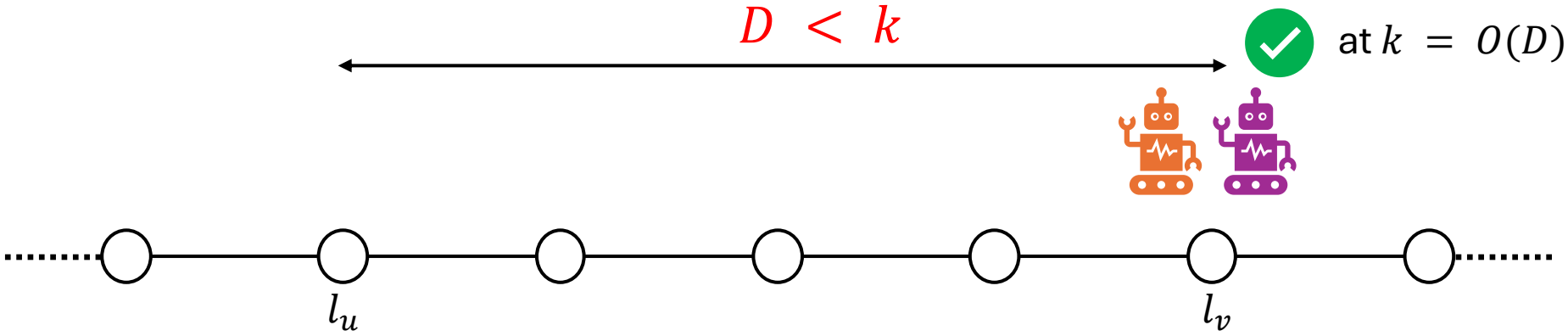

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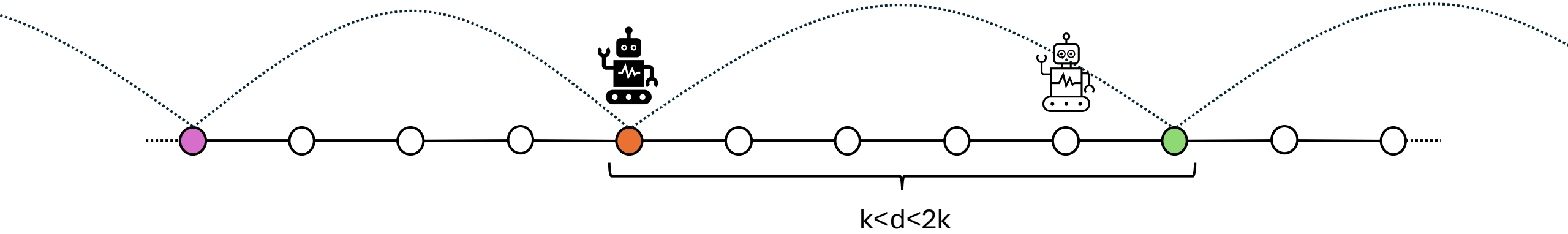
Runtime: **Searching Phase** **+** **Coloring Phase**

$O(D)$ Why? Color palette = constant

Idea for coloring agents

Colored ruling sets!!



$S_k \subset V$:

- Roughly equivalently distant

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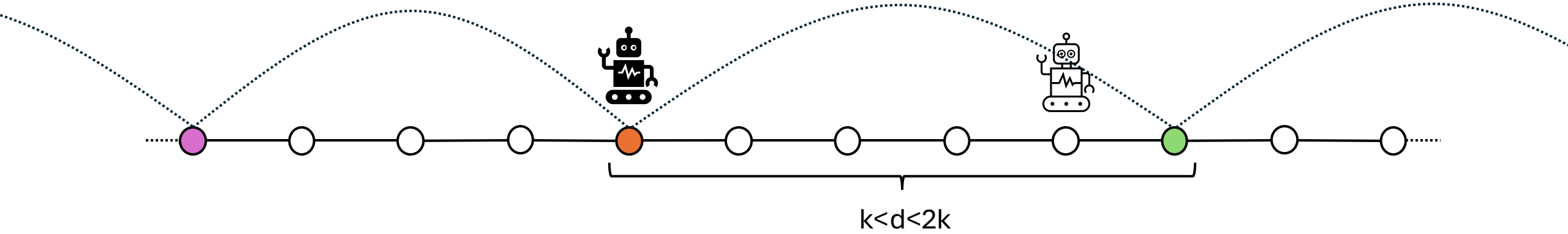

$S_k \subset V$:

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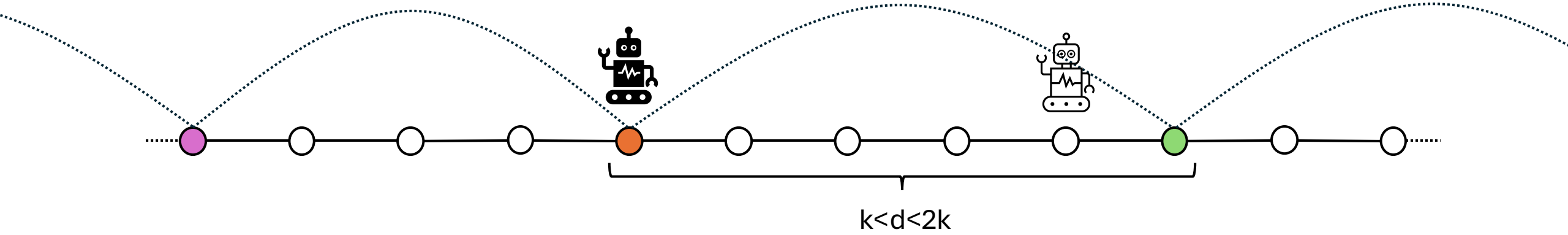

$S_k \subset V$:

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- Colored differently as nodes at distance 10k

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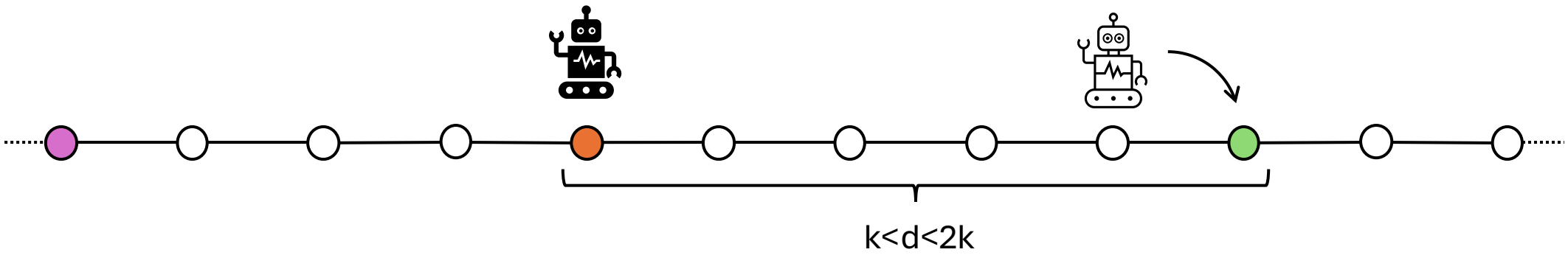

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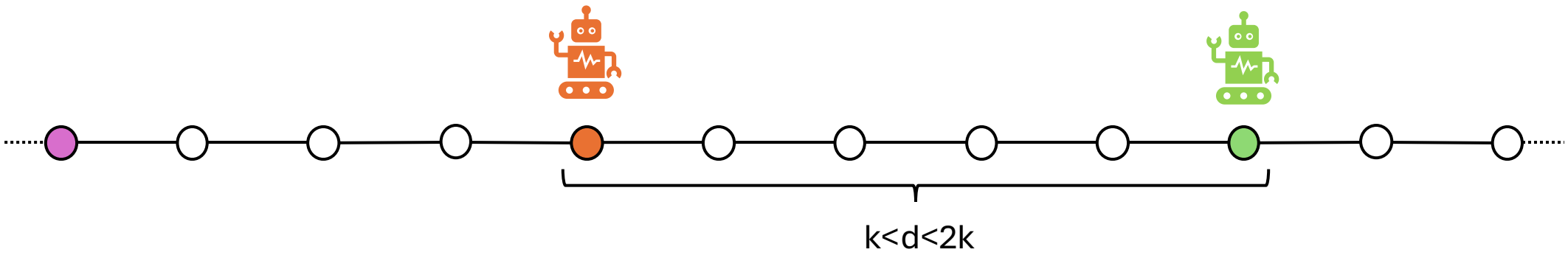

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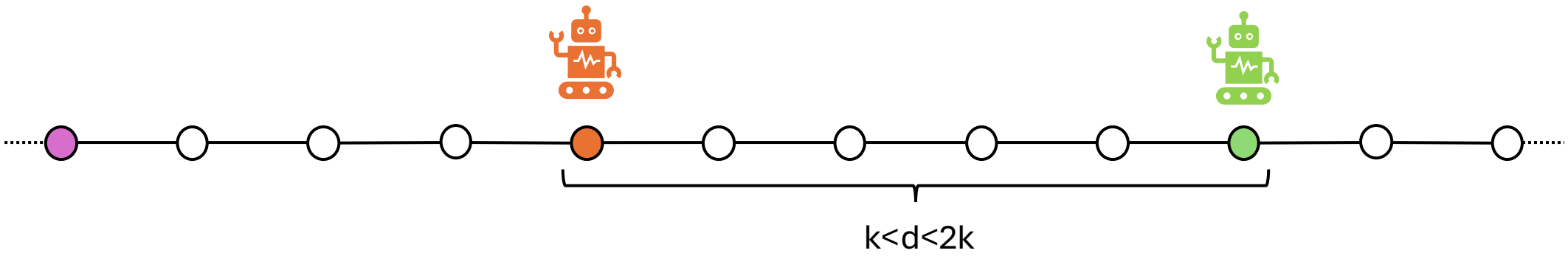

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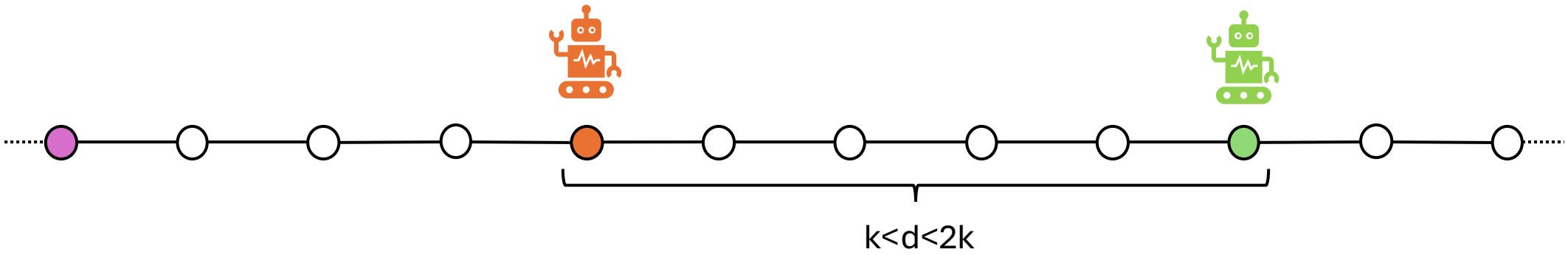
Agents can find S_k in $O(k \log^* l_{max})$ rounds

Idea for coloring agents

Colored ruling sets!!



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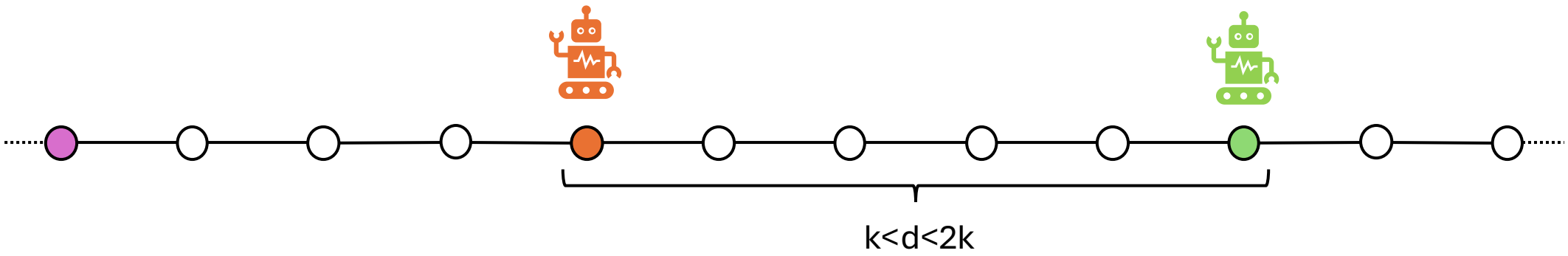
Agents can find S_k in $O(k \log^* l_{max})$ rounds

For k in {2,4,8,16,….,O(D)}

Idea for coloring agents

Colored ruling sets!!



$S_k \subset V$:

- Roughly equivalently distant
- Colored with constant size palette
- Colored differently as nodes at distance 10k

Agents can find S_k in $O(k \log^* l_{max})$ rounds

For k in {2,4,8,16,….,O(D)}

Coloring phase: $O(D \log^* l_{max})$

Running time

Runtime : $\underbrace{\textbf{Searching Phase}}_{O(D)}$ **+** $\underbrace{\textbf{Coloring Phase}}_{O(D \log^* l_{max})}$

Rendezvous in $O(D \log^* l_{max})$

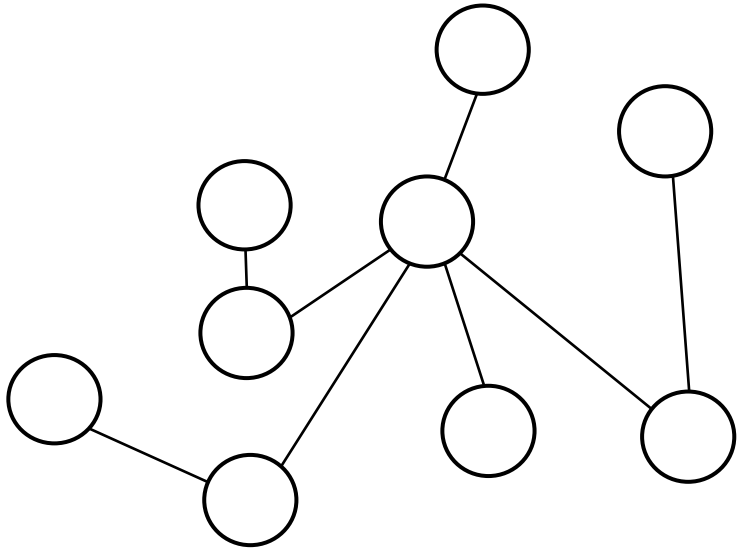
Obtaining the ruling set

Agents can find S_k in $O(k \log^* l_{max})$ rounds

Proved using another model: The LOCAL model of distributed computing

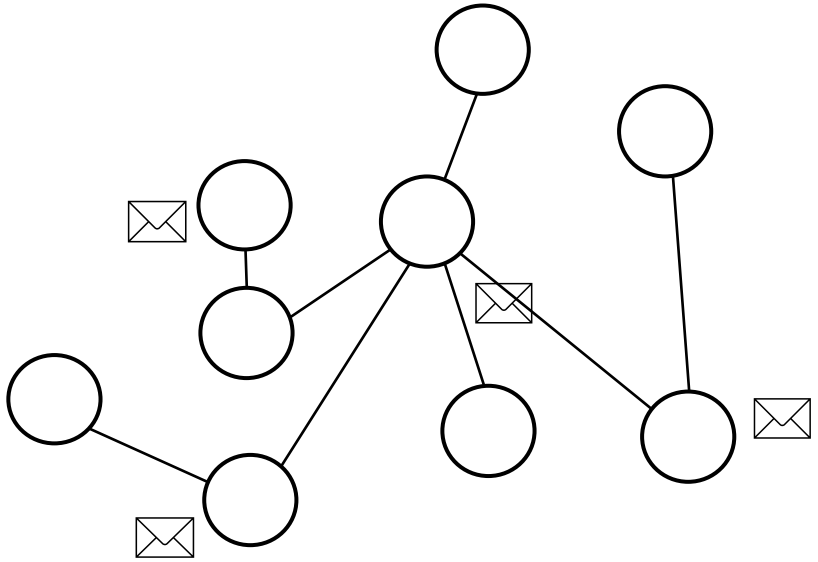
LOCAL model

- Synchronous model:
  - messages of unlimited size
  - unbounded computation
- Undirected graph G
  - unique ID
- Complexity: number of rounds until last node terminates



LOCAL model

- Synchronous model:
  - messages of unlimited size
  - unbounded computation
- Undirected graph G
  - n nodes
  - maximum degree Δ
  - unique ID
- Complexity: number of rounds until last node terminates



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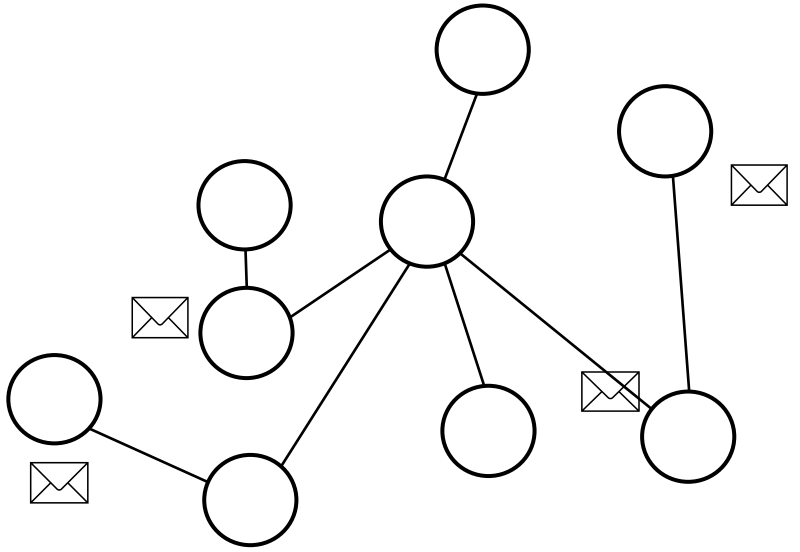

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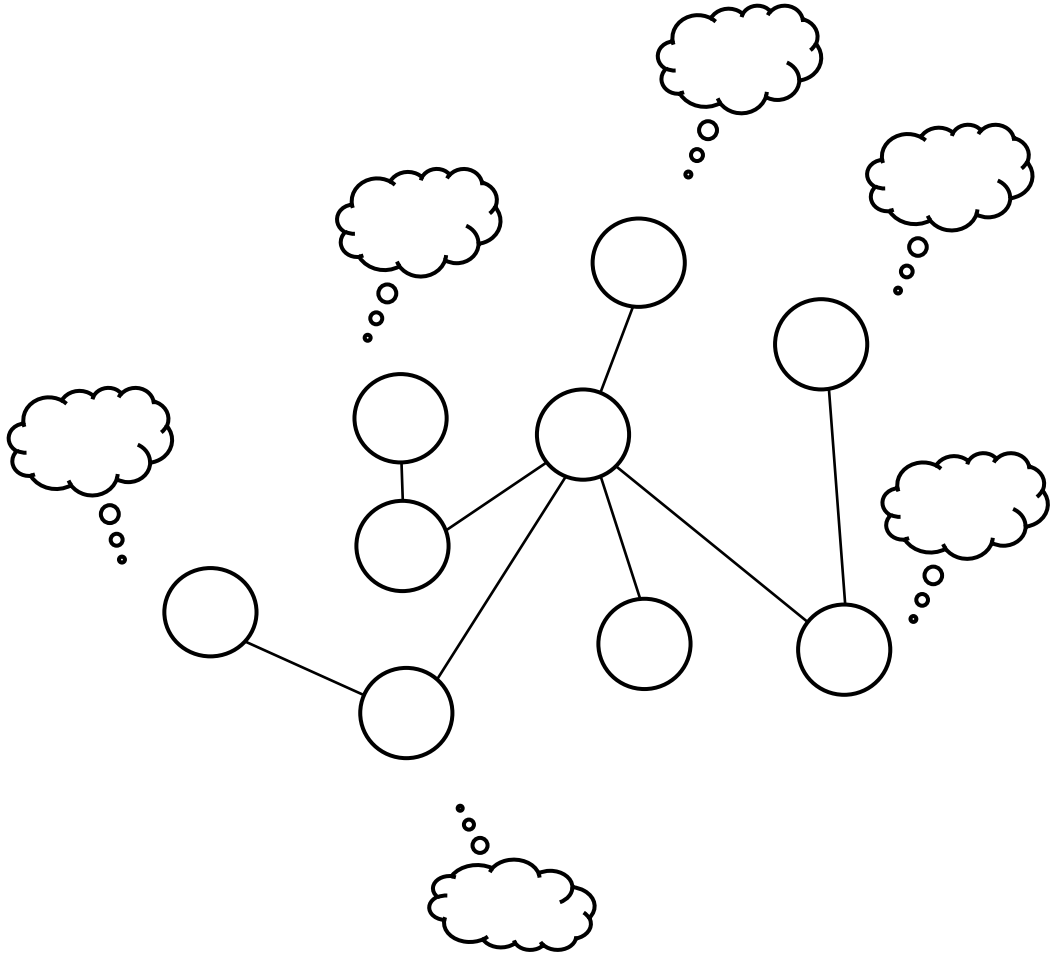
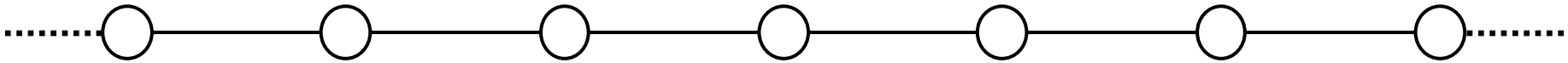


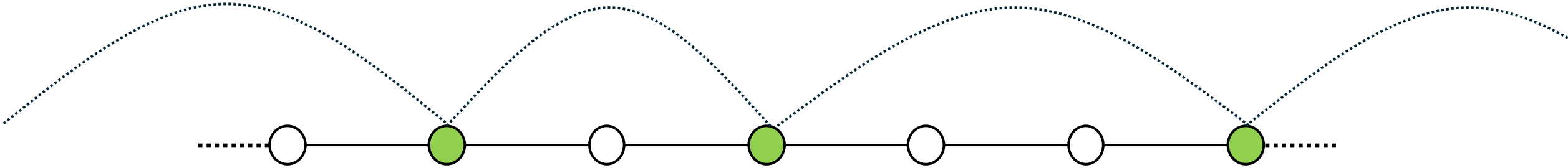
Colored ruling set in LOCAL



Colored ruling set in LOCAL

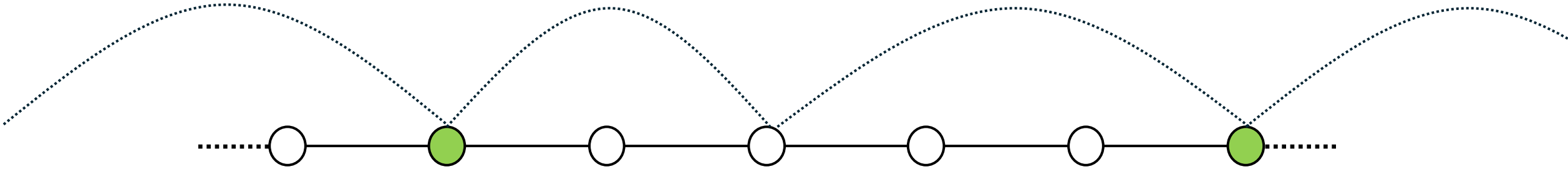
- Maximal independent set in $O(\log^* ID(v))$ rounds for every v

Colored ruling set in LOCAL



- Maximal independent set in $O(\log^* ID(v))$ rounds for every v
- Iterate on the subgraph induced by the node of the set

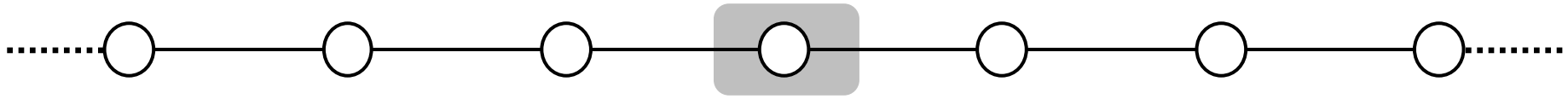
Colored ruling set in LOCAL



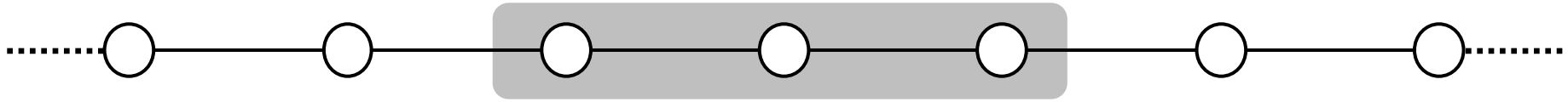
- Maximal independent set in $O(\log^* ID(v))$ rounds for every v
- Iterate on the subgraph induced by the node of the set

$$O(\mathrm{D} \log^* ID(v))$$

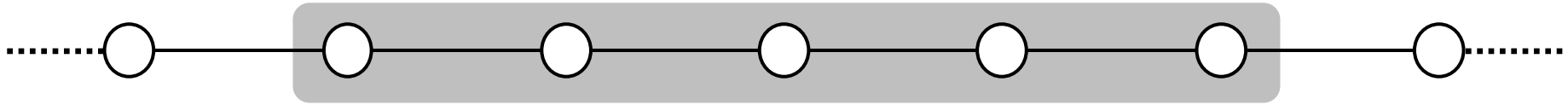
Equivalence LOCAL and agent setting



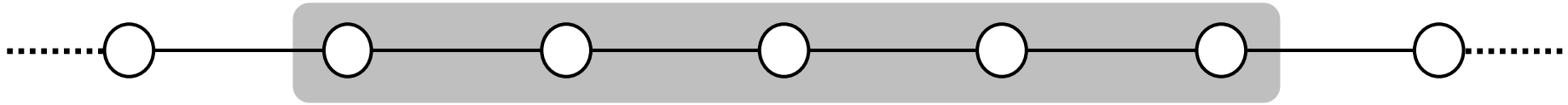
Equivalence LOCAL and agent setting



Equivalence LOCAL and agent setting



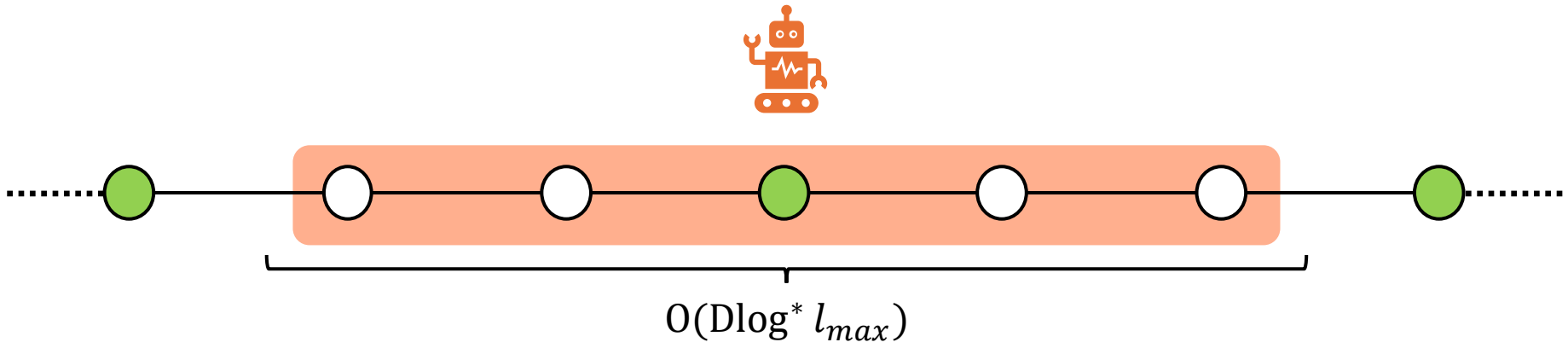
Equivalence LOCAL and agent setting



After $O(\log^* l_{max})$ rounds – sees the $O(\log^* l_{max})$-hop

Looking $O(D\log^* l_{max})$ neighborhood = messages during $O(D\log^* l_{max})$

Equivalence LOCAL and agent setting



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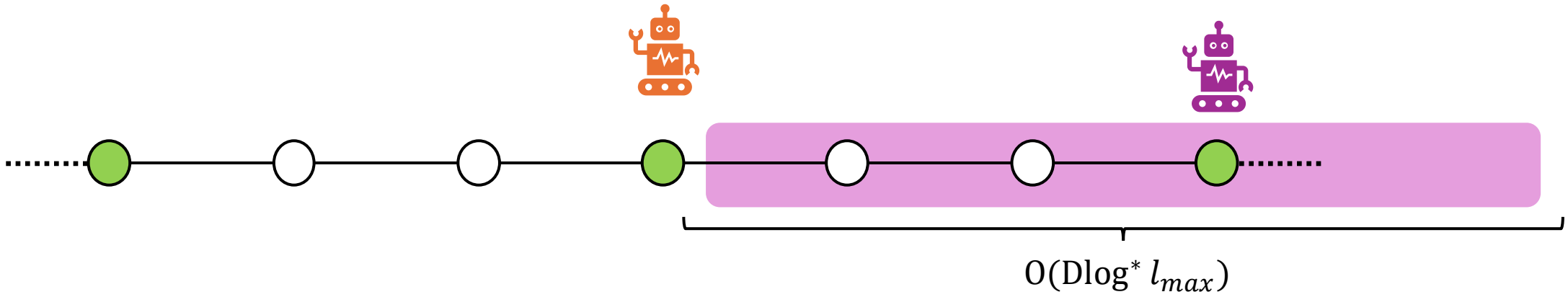

After $O(D\log^* l_{max})$ rounds – sees the $O(D\log^* l_{max})$-hop

Looking $O(D\log^* l_{max})$ neighborhood = messages during $O(D\log^* l_{max})$

Exactly the same ruling set nodes

Conclusion

$\theta(\mathrm{D}\log^* l_{max})$ —rounds algorithm for labeled lines.

Open questions:

- Labeled trees?
- Labeled grids?