

One-clock synthesis problems

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³Oxford University, UK

STACS'26

Reactive syntheses for timed systems



Build a real-time system correct by construction



Reactive syntheses for timed systems



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Reactive syntheses for timed systems



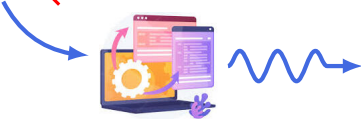
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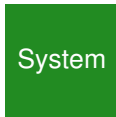
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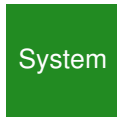
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Build a real-time system correct by construction



Input

Input: I^ω

Reactive syntheses for timed systems

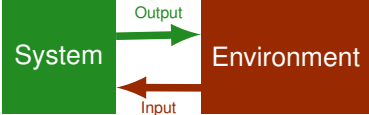
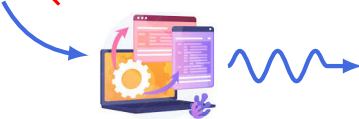


Build a real-time system correct by construction



Output: O^ω

Input: I^ω



Reactive syntheses for timed systems



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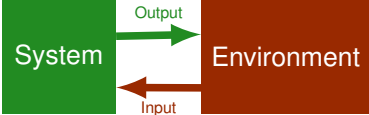


Interactions $\rightsquigarrow i_0 o_0 i_1 o_1 i_2 o_2 \dots$

Output: O^ω

Input: I^ω

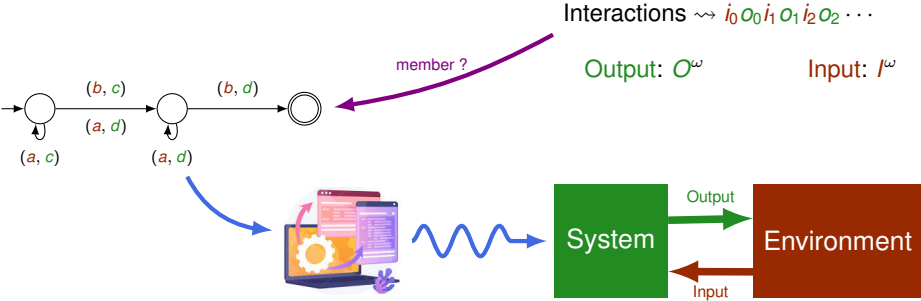
member ?



Reactive syntheses for timed systems



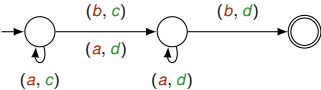
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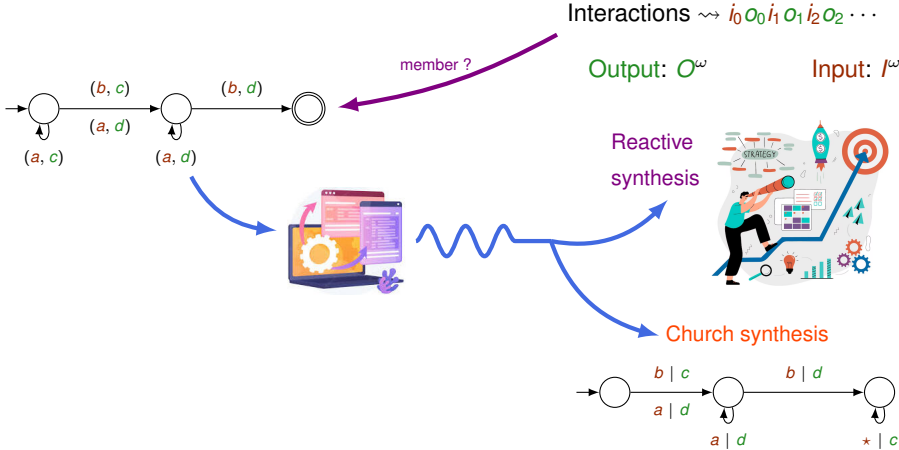
Reactive synthesis



Reactive syntheses for timed systems



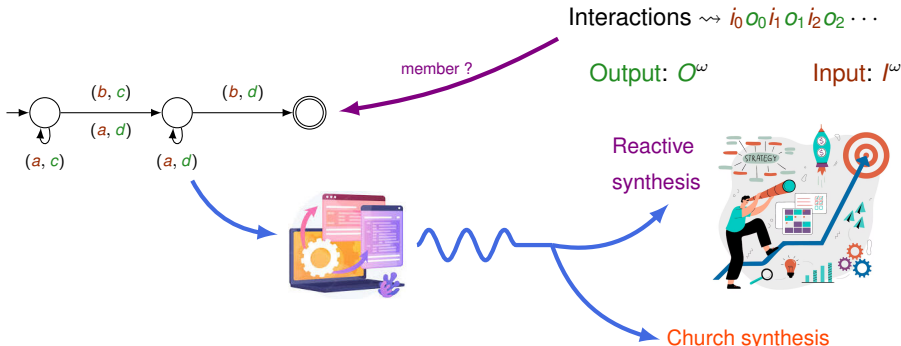
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Reactive syntheses for timed systems

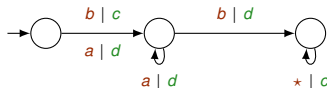


Build a real-time system correct by construction



Reactive syntheses problems

EXPTIME-complete for non-deterministic parity automata.



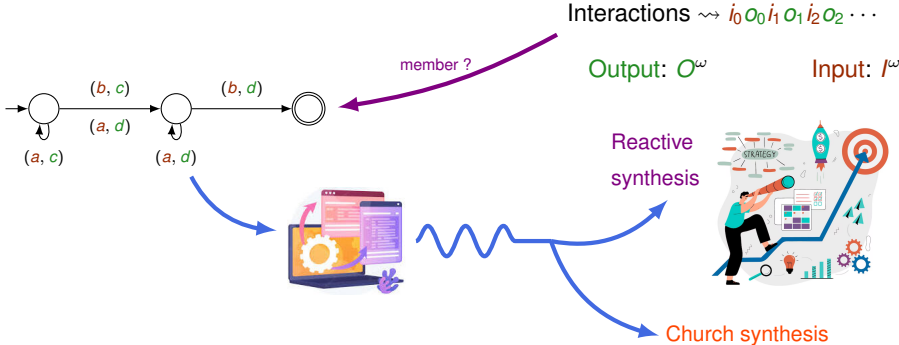
Solving sequential conditions finite-state strategies, J.R. Büchi and L.H. Landweber, 1969, AMS

On the Synthesis of a Reactive Module, A. Pnueli and R. Rosner, 1989, POPL

Reactive syntheses for timed systems

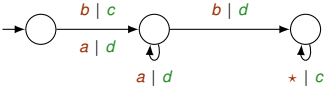


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Reactive syntheses problems

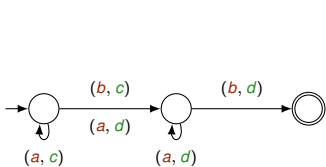
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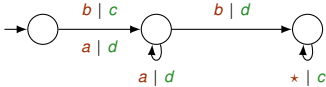
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Church synthesis

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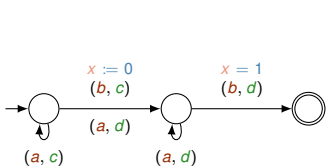
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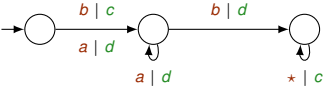
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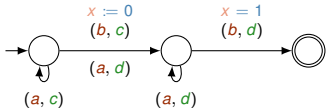


A theory of timed automata, R. Alur D. Dill, TCS 1994

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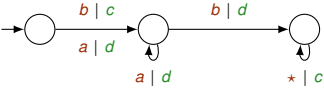
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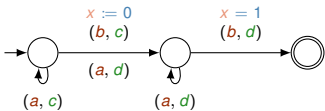
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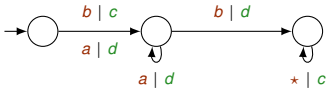


Church synthesis



Reactive syntheses problems

EXPTIME-complete for non-deterministic parity automata.



Reactive synthesis seen as a two-player game

How to play in this game?



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(a_1, t_1)

Reactive synthesis seen as a two-player game

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(a_1, t_1)

b_1

Reactive synthesis seen as a two-player game

How to play in this game?



(a_1, t_1)

(a_2, t_2)

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Reactive synthesis seen as a two-player game

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(a_1, t_1)

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b_1

b_2

Reactive synthesis seen as a two-player game

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(a_1, t_1)

(a_2, t_2)

\vdots

b_1

b_2

Reactive synthesis seen as a two-player game

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\vdots

Produce $w \in (A \times B \times \mathbb{Q}_{\geq 0})^\omega$

Reactive synthesis seen as a two-player game

How to play in this game?



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(a_2, t_2)

\vdots

b_1

b_2

\vdots

Winning condition: \mathcal{A}

a non-deterministic timed automata
over $A \times B$



wins $\Leftrightarrow w \in \mathcal{L}(\mathcal{A})$

Produce $w \in (A \times B \times \mathbb{Q}_{\geq 0})^\omega$

Reactive synthesis seen as a two-player game

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Deterministic TA defined a
timed game on an arena

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Reactive synthesis seen as a two-player game

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Deterministic TA defined a
timed game on an arena

Reactive syntheses on DTA

Problems are decidable over
deterministic timed automata.

Two distinct classes of reactive syntheses

Input : a NTA, the owner of the winning condition, and one player

Reactive synthesis

Church synthesis

Two distinct classes of reactive syntheses

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Output : if the player wins the game

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4 distinct problems

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4 distinct problems

- ▶ NTA are not complementable

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4 distinct problems

- ▶ NTA are not complementable
- ▶ Timed language are not Borel (a priori)



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Church synthesis

Output : if the player wins the game with a controller

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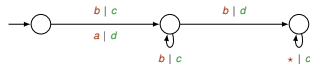


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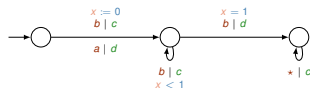


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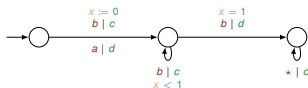


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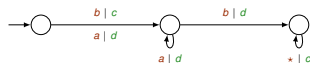
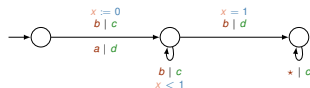


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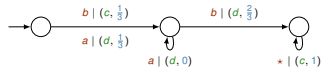
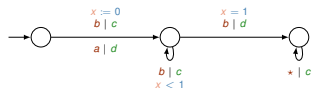


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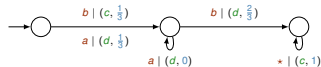
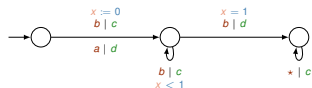


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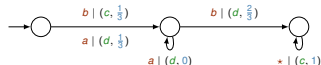
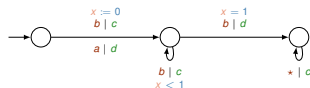
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Study the decidability of these 8 problems.





Undecidability of syntheses problems for Büchi NTA_1

		
Reactive synthesis		
Church synthesis		
		

Undecidability of syntheses problems for Büchi NTA_1

		
Reactive synthesis	Universality	
Church synthesis		Universality sampled languages
		Universality

Undecidability of syntheses problems for Büchi NTA_1





		
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For NTA_2
Reduction from
finiteness in lossy
counters machines



Undecidability of syntheses problems for Büchi NTA_1

		
Reactive synthesis	Universality	
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For NTA_2
Reduction from
finiteness in lossy
counters machines



Can we adapt this reduction for one-clock setting ?

Syntheses from lossy counters machines

Finitness in lossy counter machine

Input : \mathcal{M} a lossy counter machine

Syntheses from lossy counters machines

Finitness in lossy counter machine

Input : \mathcal{M} a lossy counter machine

(counters can always decrease by losing values)

Syntheses from lossy counters machines

Finiteness in lossy counter machine

Input : \mathcal{M} a lossy counter machine

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Output : if the set of reachable configurations is finite.

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Undecidable

Syntheses from lossy counters machines

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Timed words encode runs of \mathcal{M}

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$$\text{run: } (s_0, \nu_0) \xrightarrow{l(c_1)} (s_1, \nu_1) \xrightarrow{l(c_2)} (s_2, \nu_2) \xrightarrow{l(c_1)} (s_3, \nu_3) \xrightarrow{l(c_1)} (s_4, \nu_4) - \dots$$

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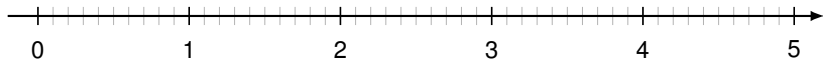
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encoding:



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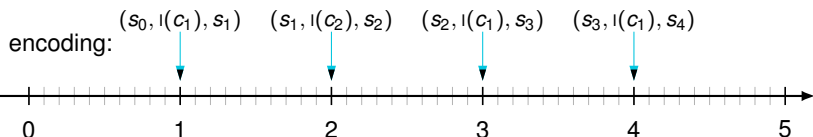


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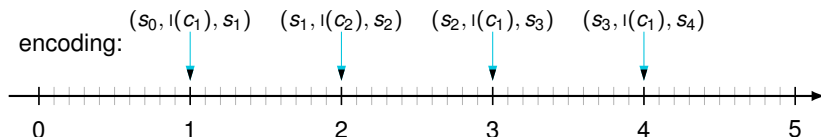


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ν_i : $\nu_0 = (0,0,0,0)$ $\nu_1 = (1,0,0,0)$ $\nu_2 = (1,1,0,0)$ $\nu_3 = (2,1,0,0)$ $\nu_4 = (0,1,0,0)$

Syntheses from lossy counters machines

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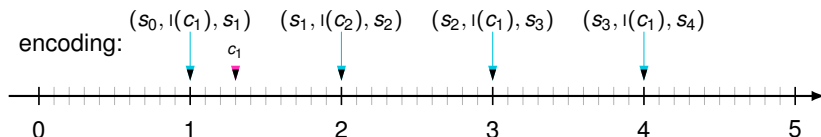


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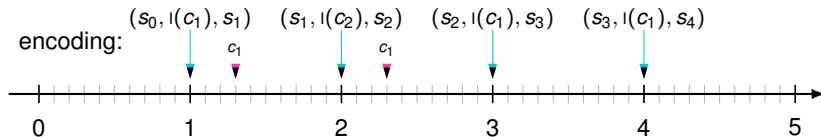


Undecidable



Timed words encode runs of \mathcal{M}

run: $(s_0, \nu_0) \xrightarrow{l(c_1)} (s_1, \nu_1) \xrightarrow{l(c_2)} (s_2, \nu_2) \xrightarrow{l(c_1)} (s_3, \nu_3) \xrightarrow{l(c_1)} (s_4, \nu_4) - \dots$



ν_i : $\nu_0 = (0,0,0,0)$ $\nu_1 = (1,0,0,0)$ $\nu_2 = (1,1,0,0)$ $\nu_3 = (2,1,0,0)$ $\nu_4 = (0,1,0,0)$

Syntheses from lossy counters machines

Finiteness in lossy counter machine

Input : \mathcal{M} a lossy counter machine

(counters can always decrease by losing values)

Output : if the set of reachable configurations is finite.

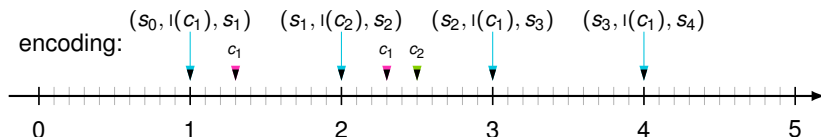


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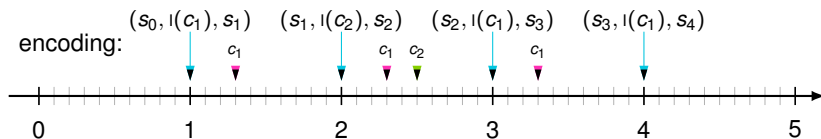


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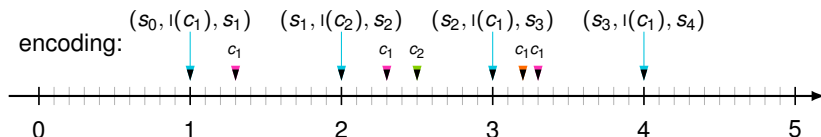


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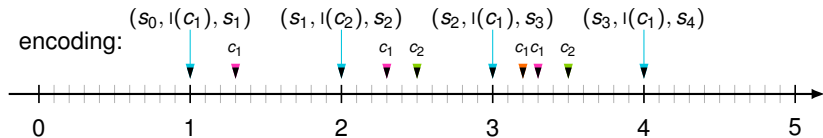


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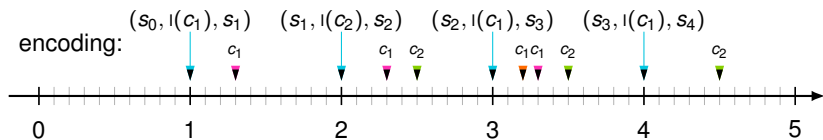


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Timed words encode
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Undecidable



Timed words encode
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\mathcal{M} a LCM

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Undecidable



Timed words encode runs of \mathcal{M}

\mathcal{M} a LCM



Reactive synthesis game \mathcal{G}

actions



encoding a run or almost a run

Syntheses from lossy counters machines

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Undecidable



Timed words encode runs of \mathcal{M}

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encoding a run or almost a run



monitor the run to detect errors

Syntheses from lossy counters machines

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Undecidable

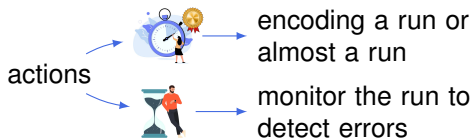


Timed words encode runs of \mathcal{M}


\mathcal{M} a LCM



Reactive synthesis game \mathcal{G}



winning condition : NTA_2

if  fails in monitoring

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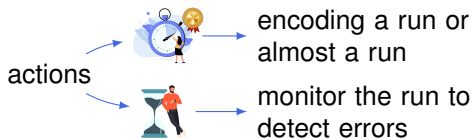


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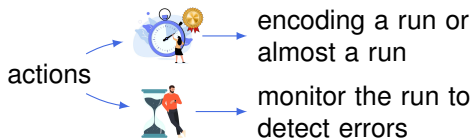


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monitor the run to detect errors

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Why two clocks are needed ?

Removing one clock in the winning condition



can win by encoding a good run of \mathcal{M}

Removing one clock in the winning condition



can win by encoding a good run of \mathcal{M}

Büchi in lossy counter machine

Input : \mathcal{M} a lossy counter machine and q_f

Output : if q_f is repeatedly reachable.

Removing one clock in the winning condition



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Input : \mathcal{M} a lossy counter machine and q_f

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wins with a controller



needs infinite memory to win

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Use a new encoding of runs into timed words


Removing one clock in the winning condition

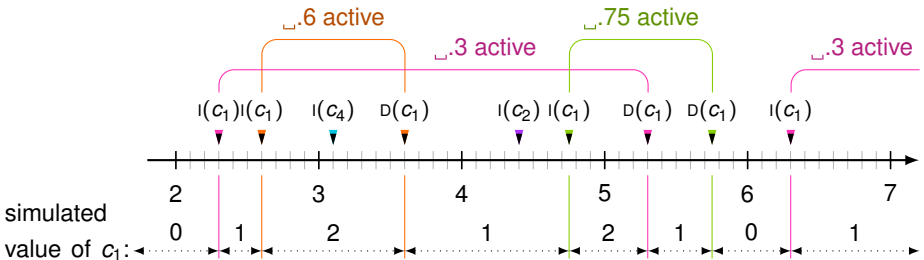

 can win by encoding a good run of \mathcal{M}

Büchi in lossy counter machine





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 wins with a controller
 needs infinite memory to win

 Use a new encoding of runs into timed words







Undecidability of syntheses problems for Büchi NTA_1

		
Reactive synthesis	Universality	
Church synthesis		Universality sampled languages
		Universality



For NTA_2
Reduction from
finiteness in lossy
counters machines





Undecidability of syntheses problems for Büchi NTA_1

			
Reactive synthesis	Universality	Büchi in lossy counters machines	
Church synthesis		Universality sampled languages	Büchi in lossy counters machines
		Universality	Finiteness in lossy counters machines



For NTA_2
Reduction from finitness in lossy counters machines

Undecidability of syntheses problems for Büchi NTA_1

		
Reactive synthesis	Universality	Büchi in lossy counters machines
Church synthesis		Universality sampled languages
		Universality



For NTA_2
Reduction from finiteness in lossy counters machines



Synthese of register automata

Challenging questions



Existence of decidable subclass of game ?

Challenging questions



Existence of decidable subclass of game ?



restriction of the winning condition

Challenging questions



Existence of decidable subclass of game ?

💡 restriction of the winning condition

- ▶ 1-clock reachability NTA

Challenging questions



Existence of decidable subclass of game ?



restriction of the winning condition

- ▶ 1-clock reachability NTA
- ▶ history-deterministic Büchi TA

Challenging questions



Existence of decidable subclass of game ?

- 💡 restriction of the winning condition
 - ▶ 1-clock reachability NTA
 - ▶ history-deterministic Büchi TA
- 💡 restriction of players' powers

Challenging questions



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Decidability for
resource-bounded
synthesis

Challenging questions



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change the semantics of timed automata : robustness

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-  Decidability for resource-bounded synthesis
- 💡 change the semantics of timed automata : robustness


Are theses games determined ?

Challenging questions

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Are these games determined ?

-  not Borel
- 💡 study the topology of timed languages


Challenging questions

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Thank you! Questions ?

Syntheses from universality problem in *NTA*

Universality

Input : $\mathcal{A} \in \text{NTA}$ over Σ

Output : if $\mathcal{L}(\mathcal{A}) = \mathbb{T}(\Sigma^*)$

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Undecidable

▶ reachability NTA_2

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Undecidable

▶ reachability *NTA*₂ ▶ Büchi *NTA*₁

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Reactive synthesis game \mathcal{G}

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$\mathcal{A} \in \text{NTA}$
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Reactive synthesis game \mathcal{G}

actions



Σ

Syntheses from universality problem in NTA

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$\mathcal{A} \in NTA$
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Reactive synthesis game \mathcal{G}



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winning condition : \mathcal{A}

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Reactive synthesis game \mathcal{G}



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wins $\Leftrightarrow \exists w \notin \mathcal{L}(\mathcal{A})$

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wins with a controller $\Leftrightarrow \forall w \in \mathcal{L}(\mathcal{A})$

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Reactive synthesis game \mathcal{G}



winning condition : \mathcal{A}



How to guarantee a
winning controller for



wins $\Leftrightarrow \exists w \notin \mathcal{L}(\mathcal{A})$



wins with a controller $\Leftrightarrow \forall w \in \mathcal{L}(\mathcal{A})$

Controller for from universality problems

Universal sampled timed language

Input : \mathcal{A} a NTA

Output : if for all $\varepsilon > 0$, $\mathcal{L}_\varepsilon(\mathcal{A})$ is universal.

Controller for from universality problems

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Controller for from universality problems

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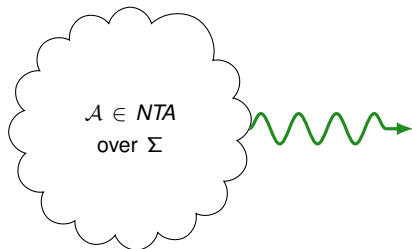
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Reactive synthesis game \mathcal{G}



winning condition : \mathcal{A}

Controller for from universality problems

Universal sampled timed language

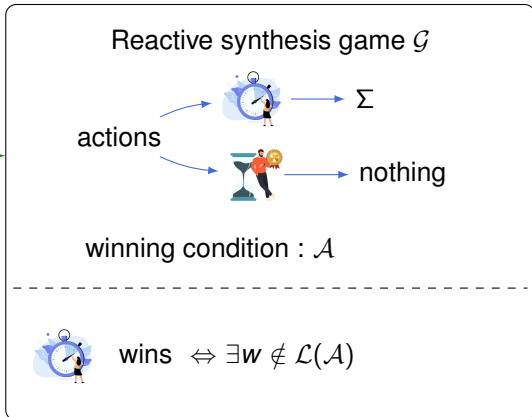
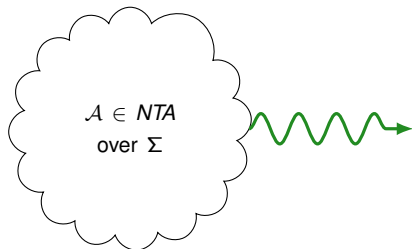
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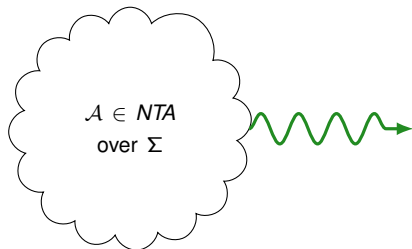
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Undecidable

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Reactive synthesis game \mathcal{G}



winning condition : \mathcal{A}



wins with a controller \Leftrightarrow

$\exists \varepsilon, w \mid w \in \mathcal{L}_\varepsilon(\Sigma^\omega) \setminus \mathcal{L}_\varepsilon(\mathcal{A})$

Controller for from universality problems

Universal sampled timed language

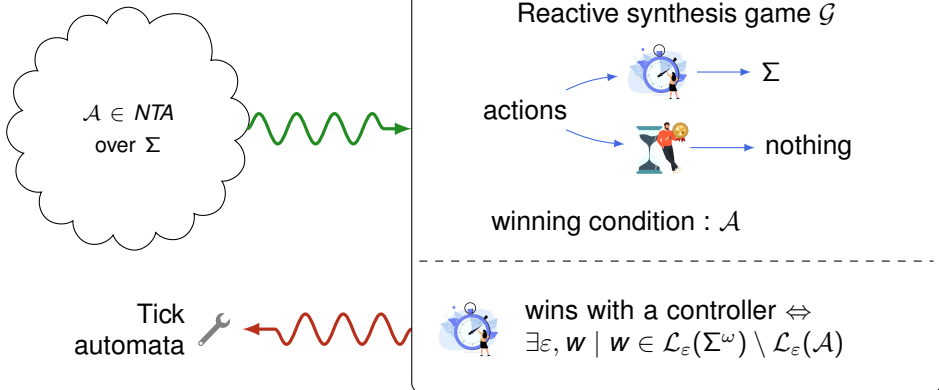
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Undecidable

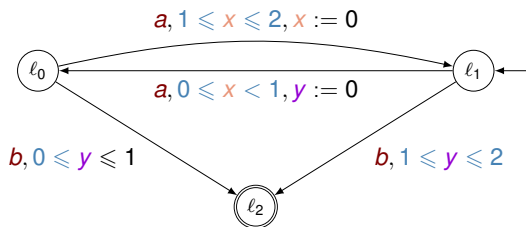
► Büchi NTA₁



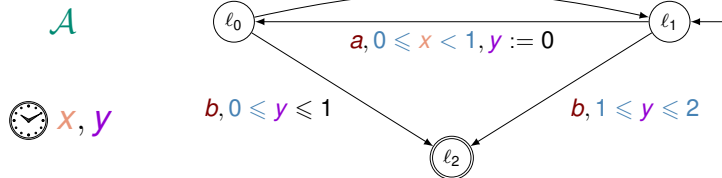
On timed automata with discrete time – structural and language theoretical characterization, H. Gruber, M. Holzer, A. Kiehn, and B. König, DLT, 2005.

Interlude : focus on timed automata

A



Interlude : focus on timed automata

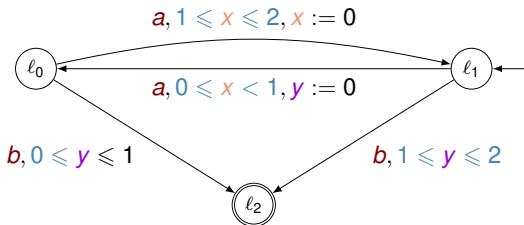


Interlude : focus on timed automata

\mathcal{A}



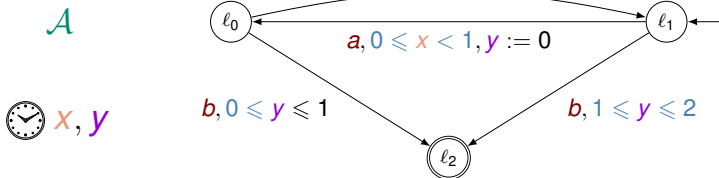
x, y



Language of \mathcal{A}

$w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

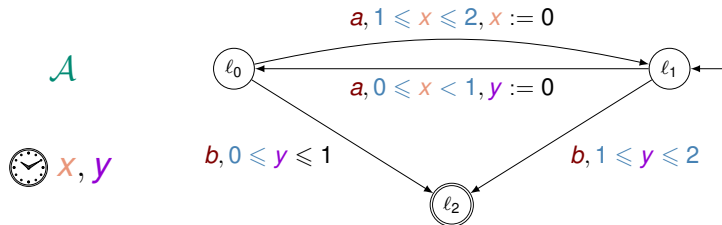
Interlude : focus on timed automata



Language of \mathcal{A}

$(a, 0.5)(a, 1.25)(b, \frac{1}{3}) = w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

Interlude : focus on timed automata

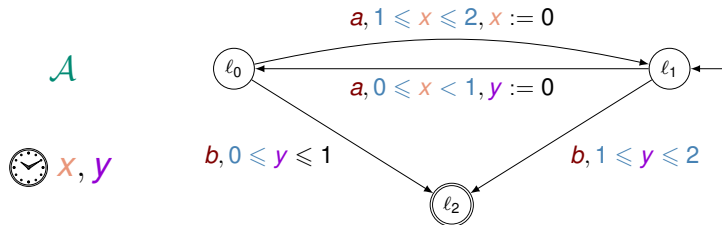


Language of \mathcal{A}

$(a, 0.5)(a, 1.25)(b, \frac{1}{3}) = w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

$$(\ell_1, \begin{bmatrix} x \mapsto 0 \\ y \mapsto 0 \end{bmatrix})$$

Interlude : focus on timed automata

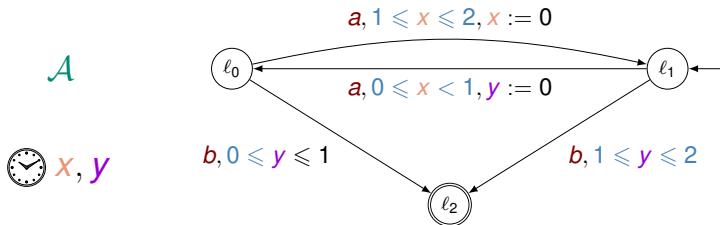


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$$(l_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$$

Interlude : focus on timed automata

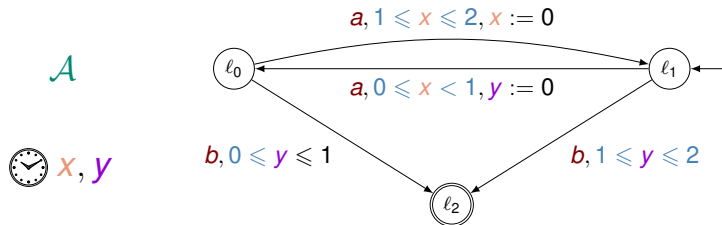


Language of \mathcal{A}

$(a, 0.5)(a, 1.25)(b, \frac{1}{3}) = w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

$$(l_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{a, 0.5}$$

Interlude : focus on timed automata

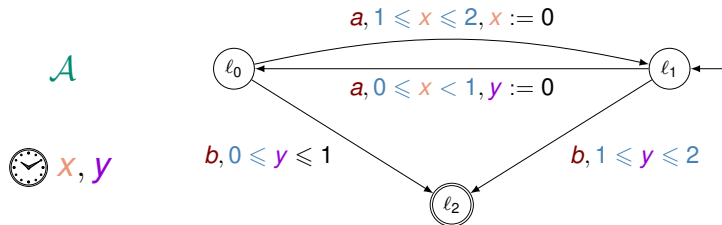


Language of \mathcal{A}

$(a, 0.5)(a, 1.25)(b, \frac{1}{3}) = w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

$$(l_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{a, 0.5} (l_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix})$$

Interlude : focus on timed automata

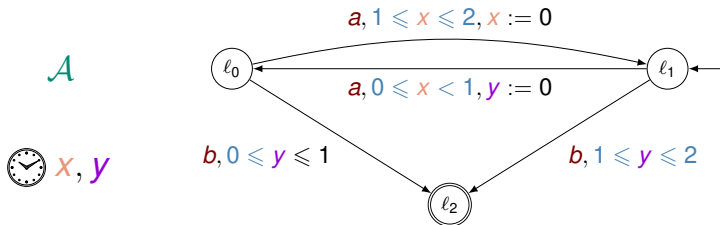


Language of \mathcal{A}

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$$(l_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{a, 0.5} (l_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{a, 1.25} (l_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{b, 1/3} (l_2, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$$

Interlude : focus on timed automata



Language of \mathcal{A}

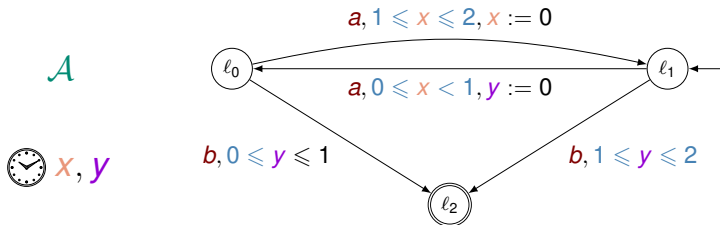
$(a, 0.5)(a, 1.25)(b, \frac{1}{3}) = w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow \exists$ an accepting execution on w in \mathcal{A}

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Wanted properties for timed automata

Interlude : focus on timed automata



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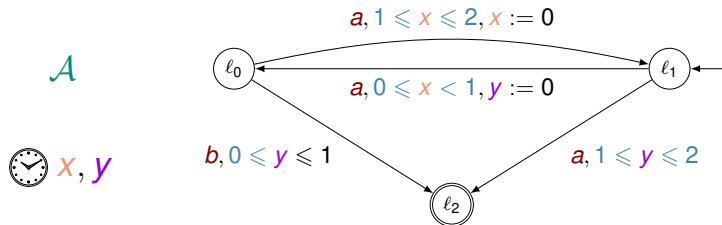


Wanted properties for timed automata



Not determinizable

Interlude : focus on timed automata



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Wanted properties for timed automata



Not determinizable



Not stable by complement