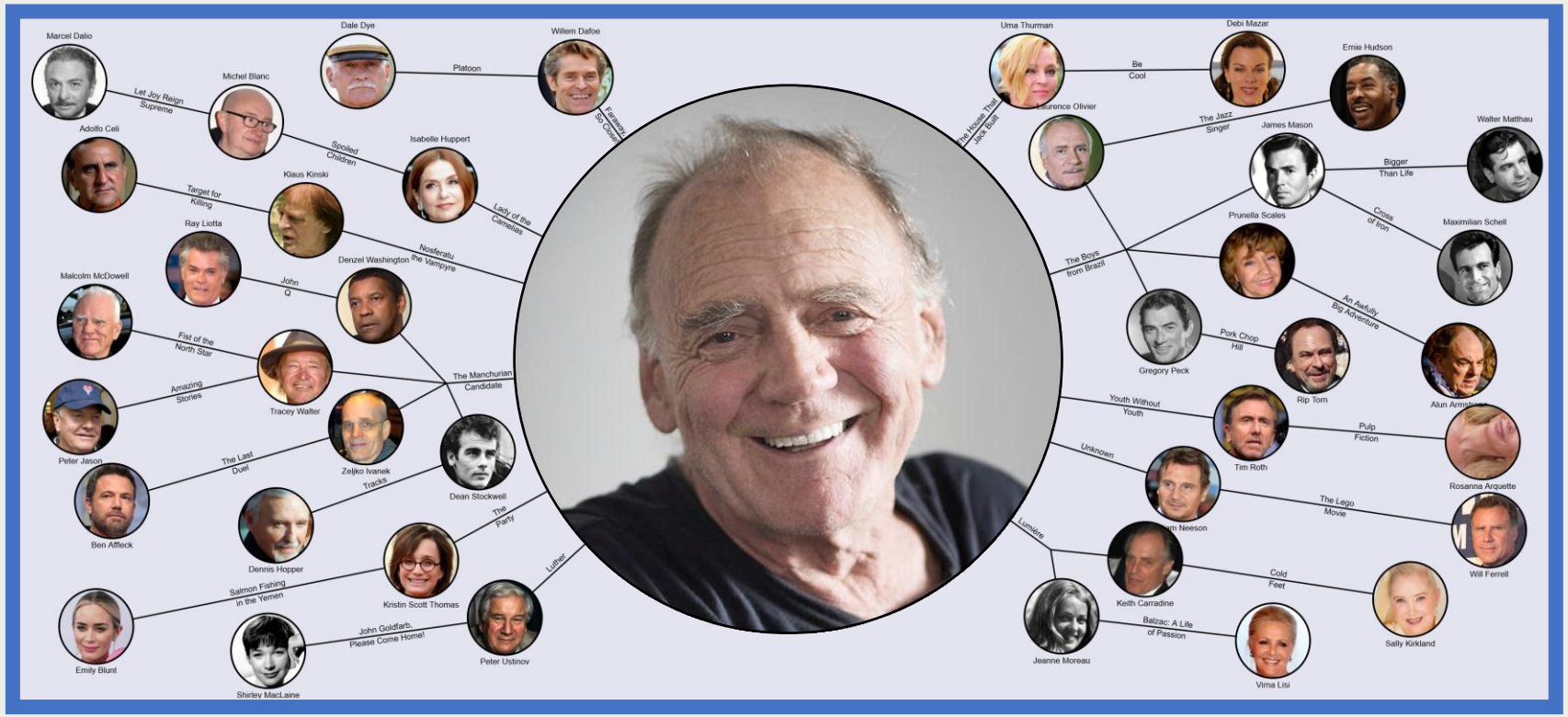
The Diameter of (Threshold) Geometric Inhomogeneous Random Graphs

Z. Benjert, K. Lakis, J. Lengler, R.R. Ravi



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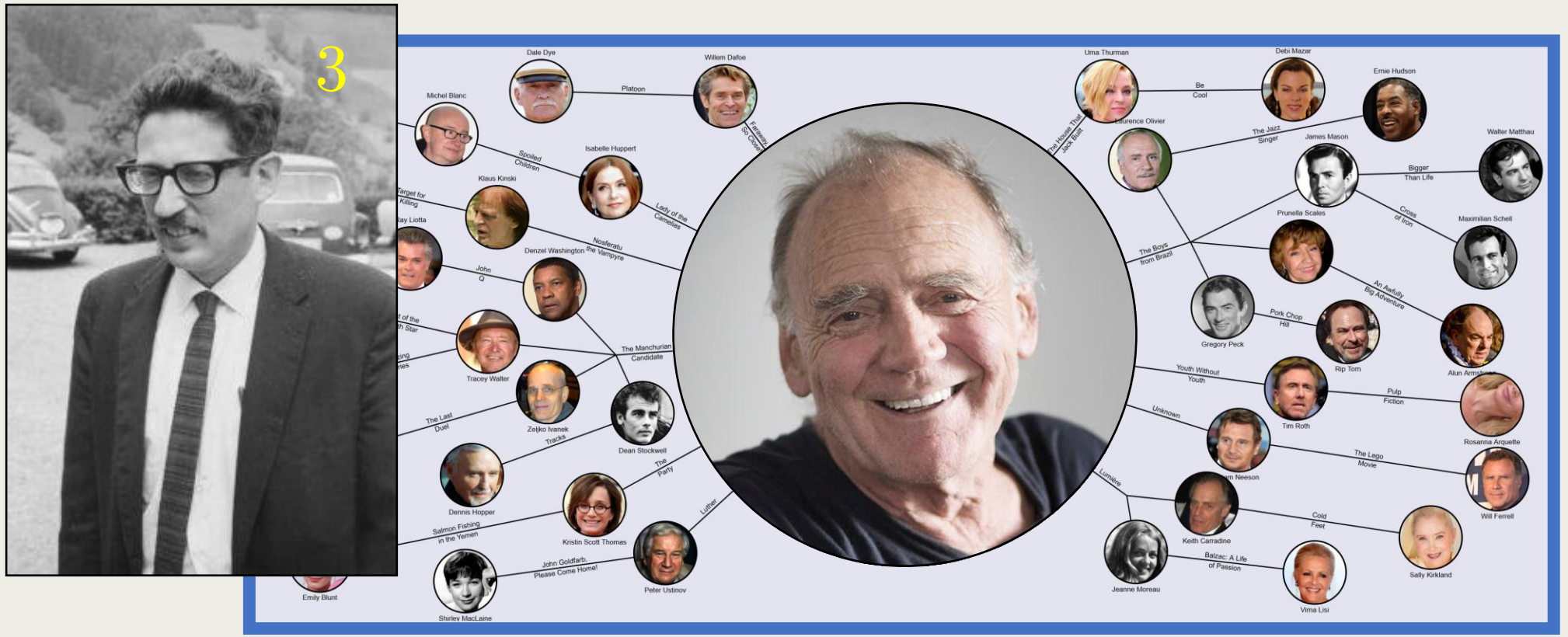

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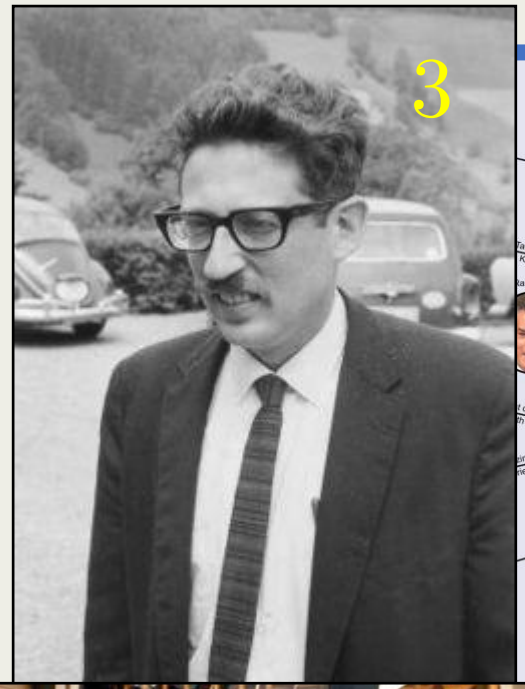

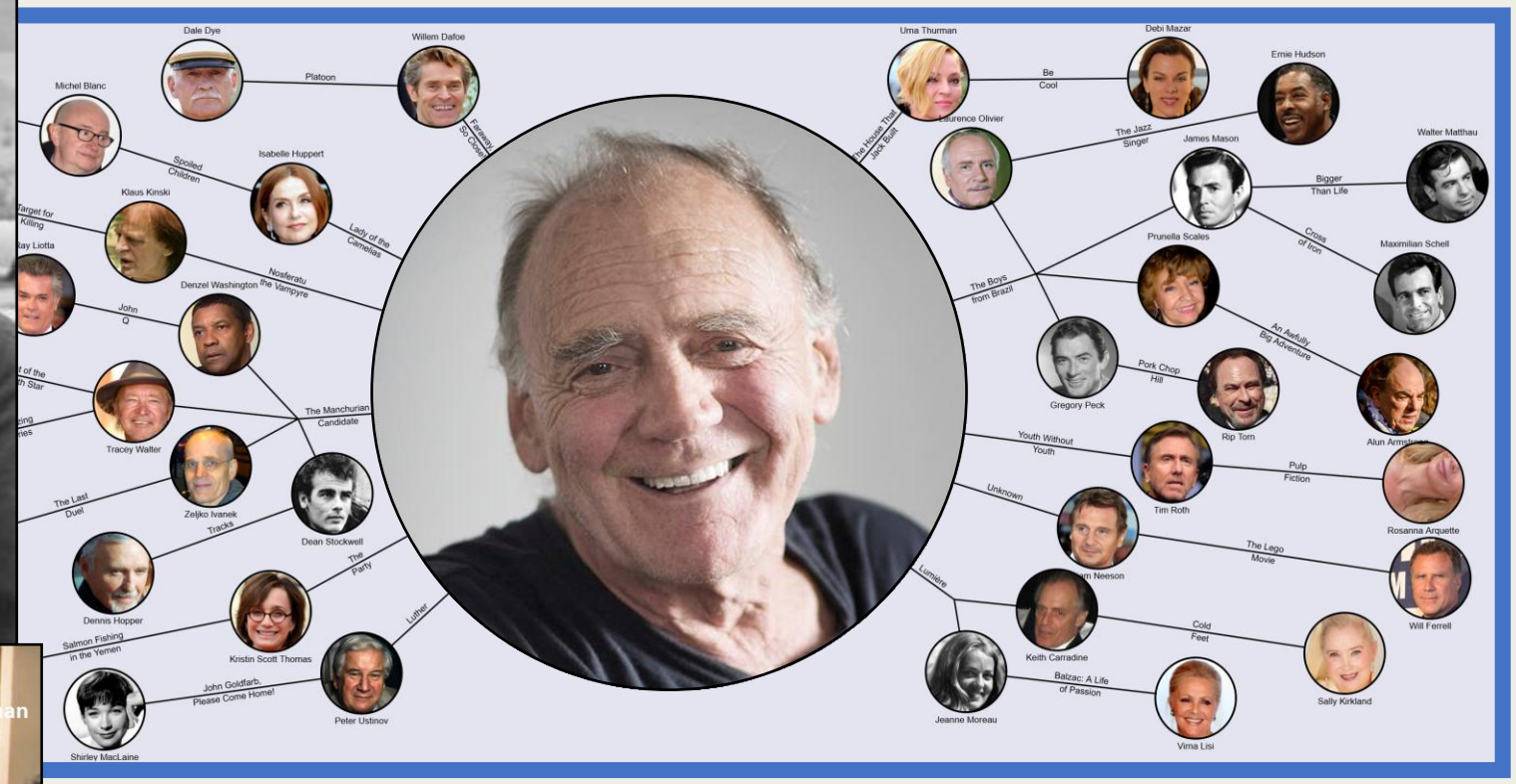

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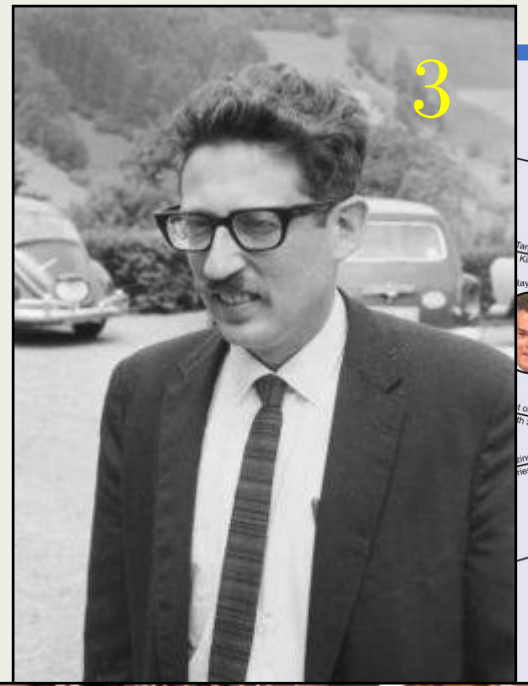

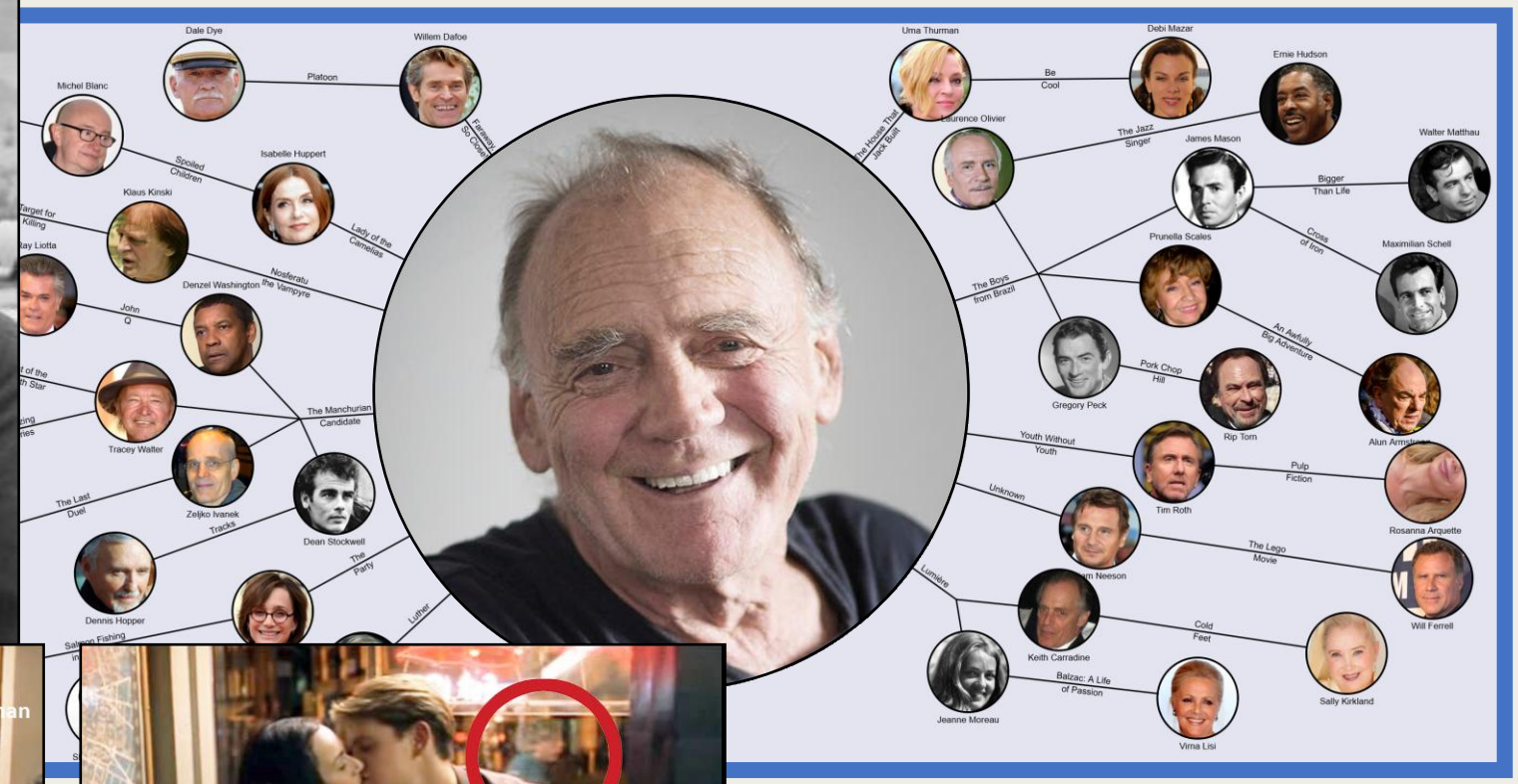

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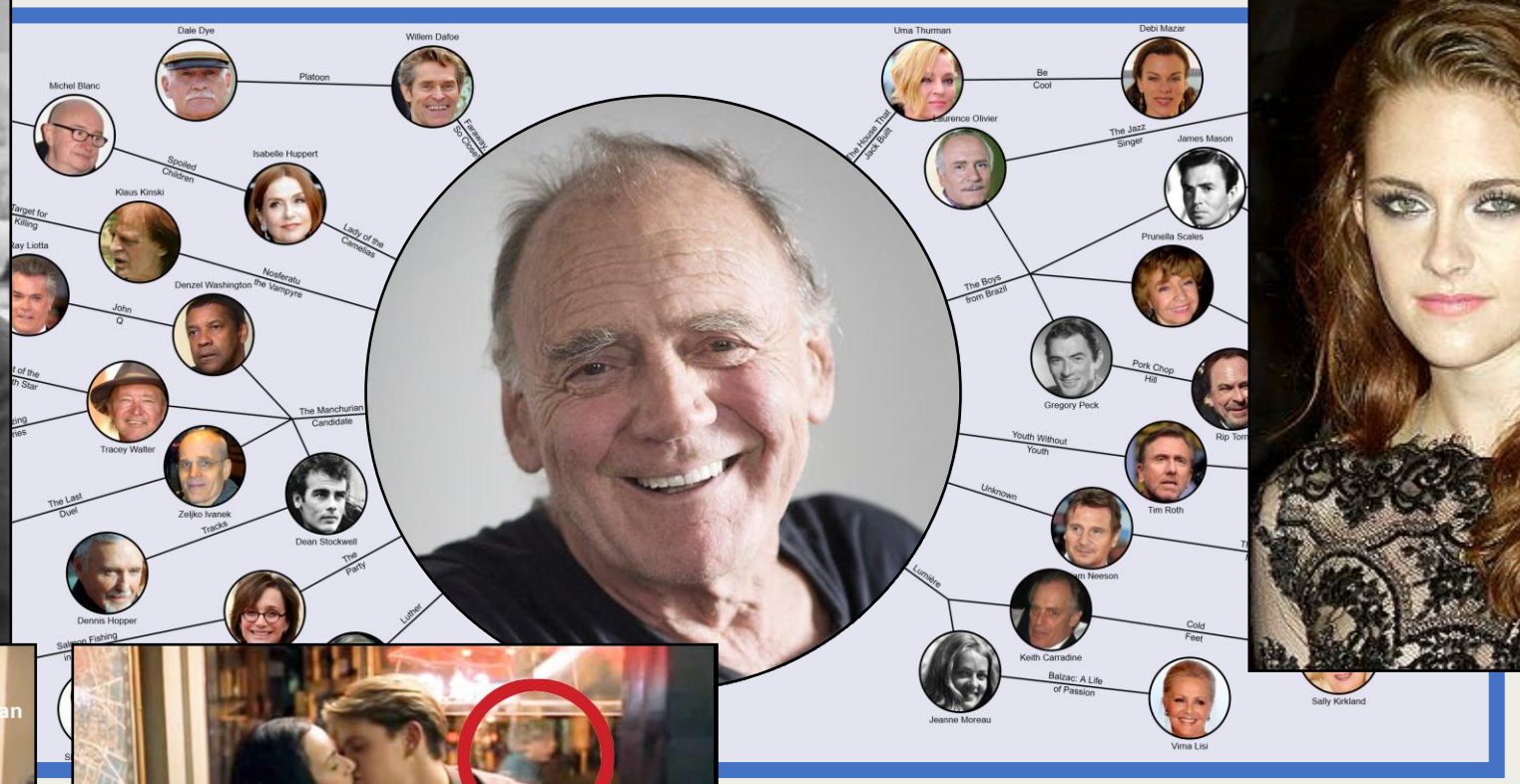

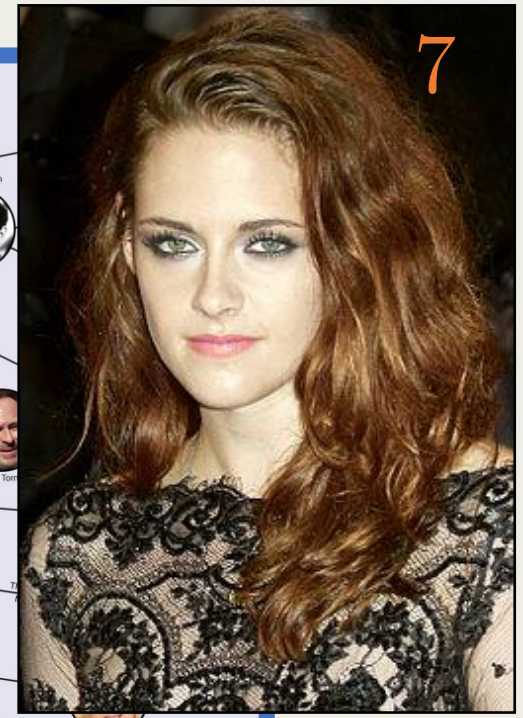

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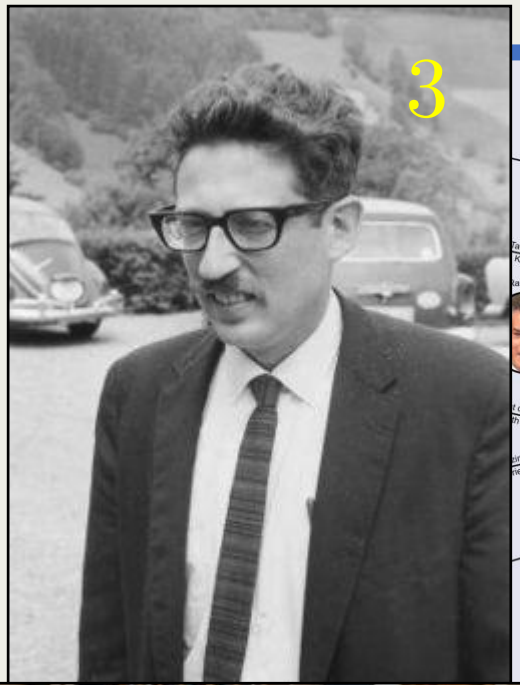

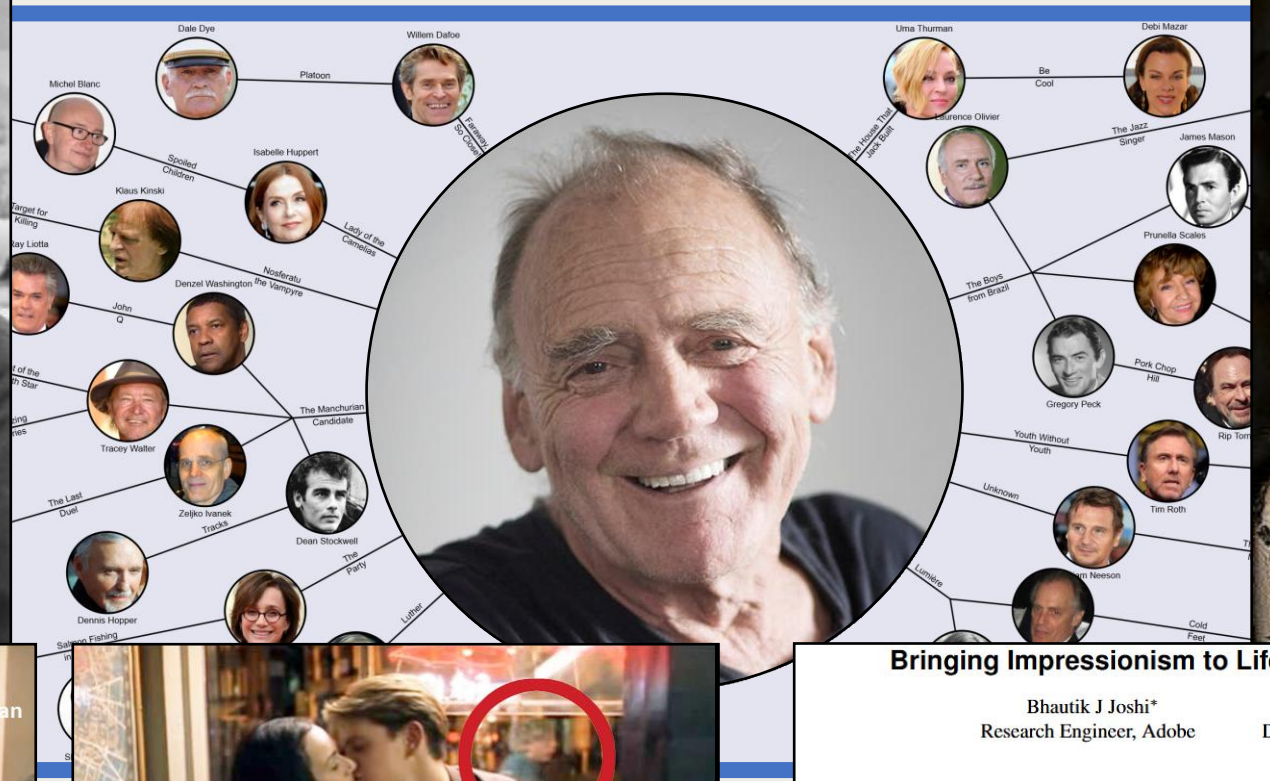

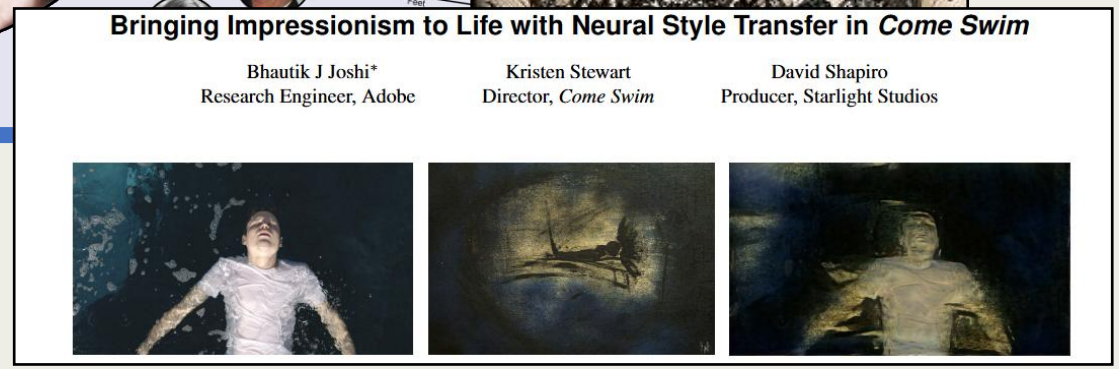

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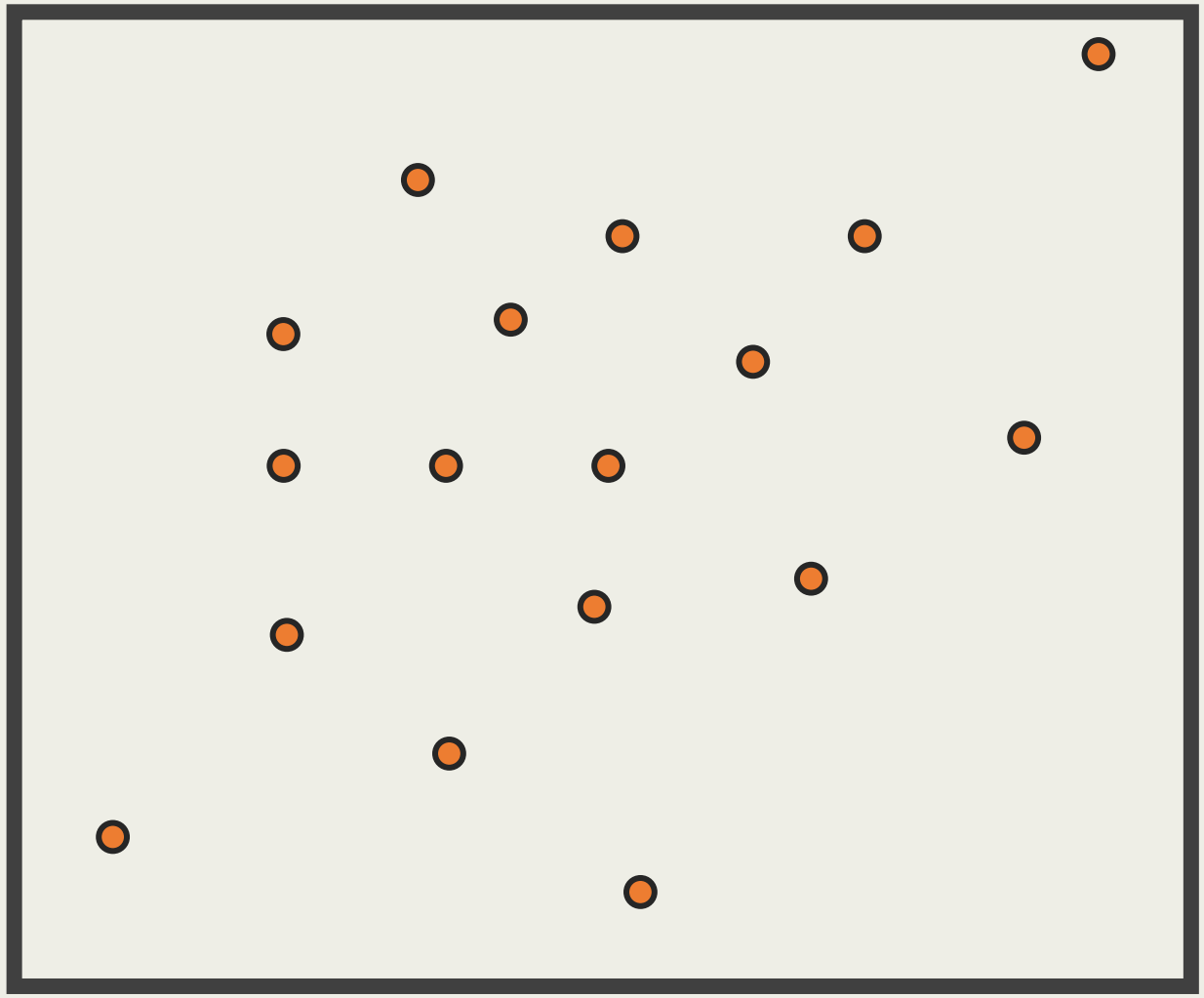

Geometric Inhomogeneous Random Graphs (GIRGs)

Power law weights:

$$P(w \geq z) = z^{1-\tau}$$

Tunable parameter τ,
inversely proportional to weights.

We consider τ between 2 and 3 for this talk.

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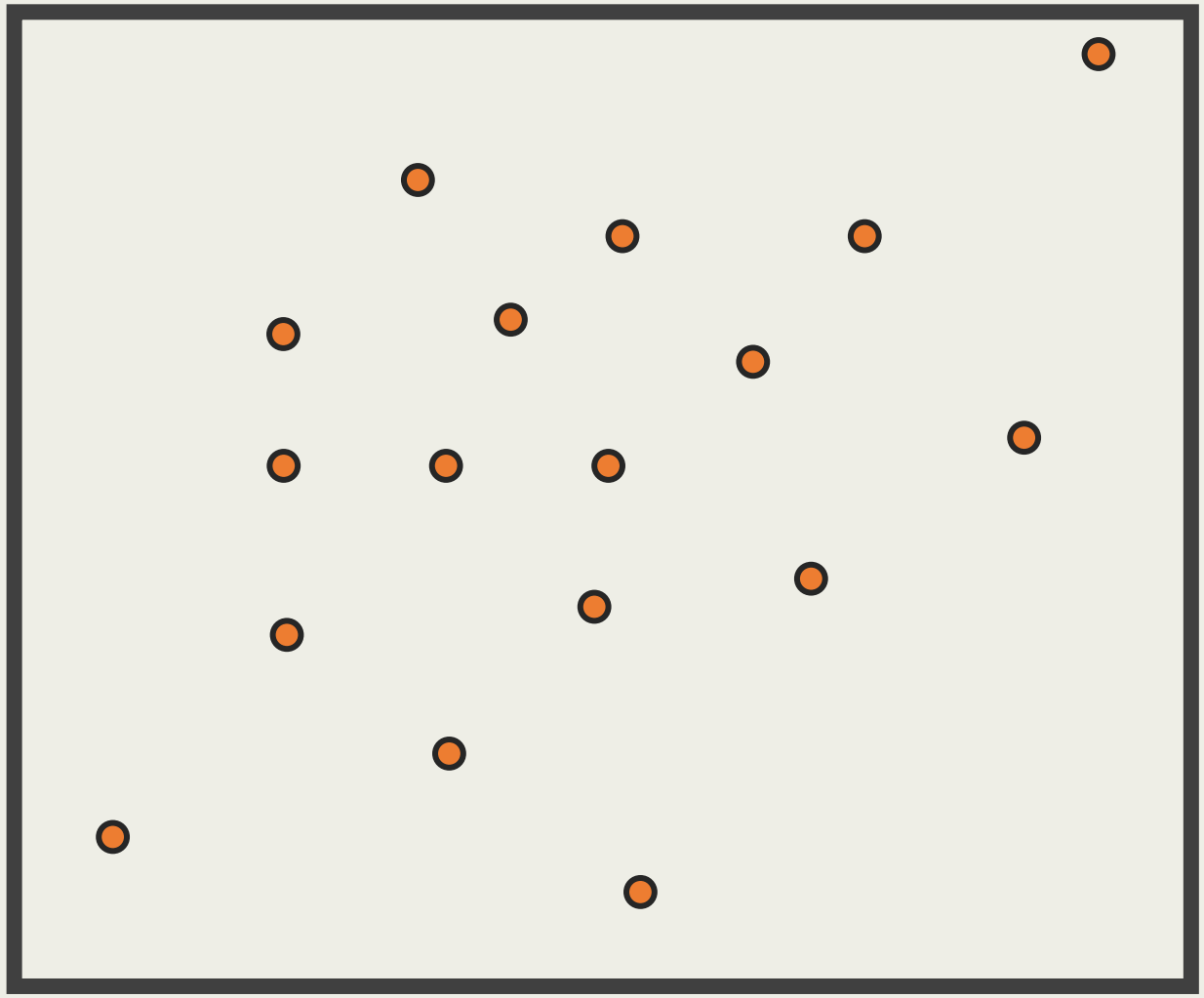

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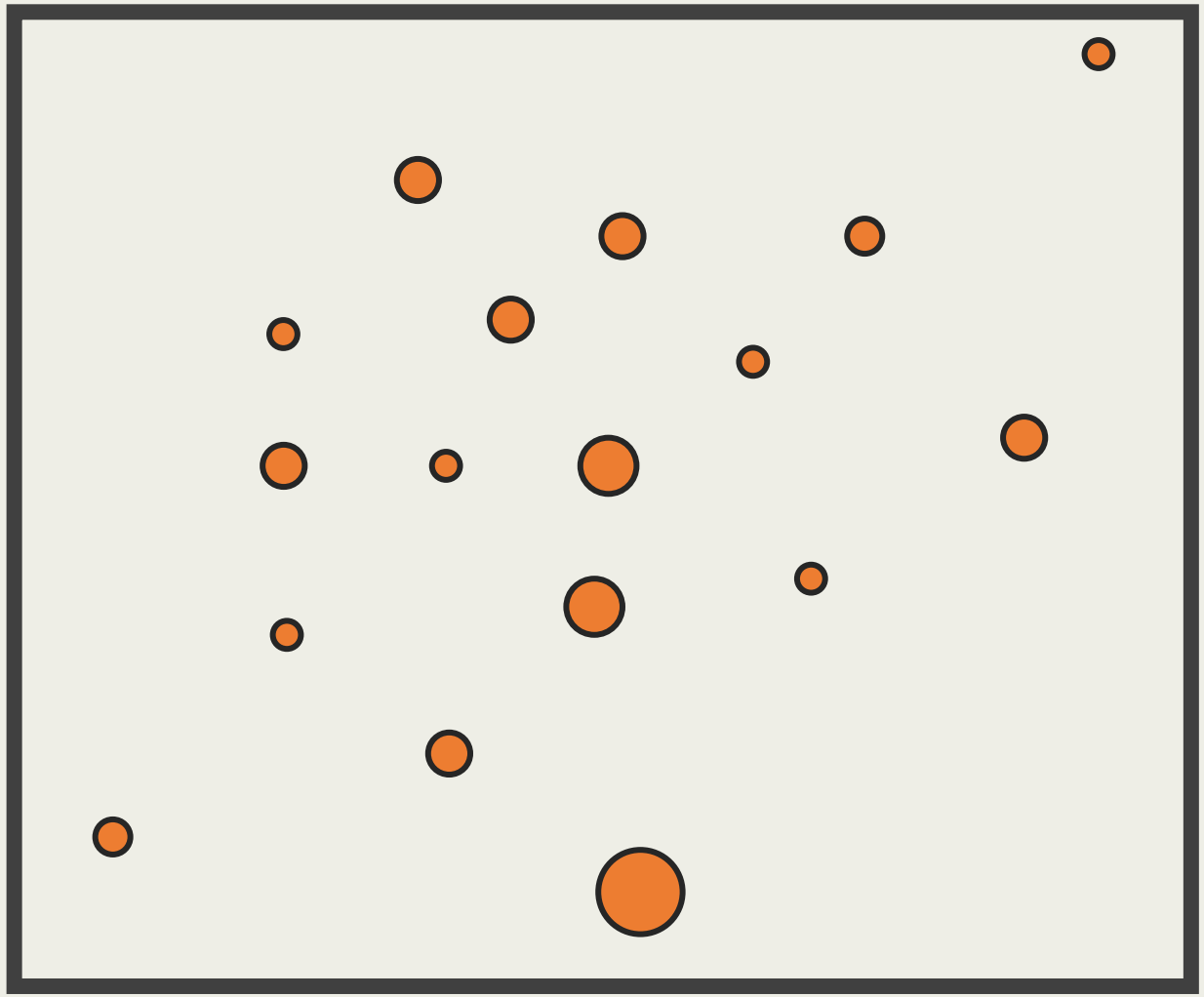

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Vertices are connected iff

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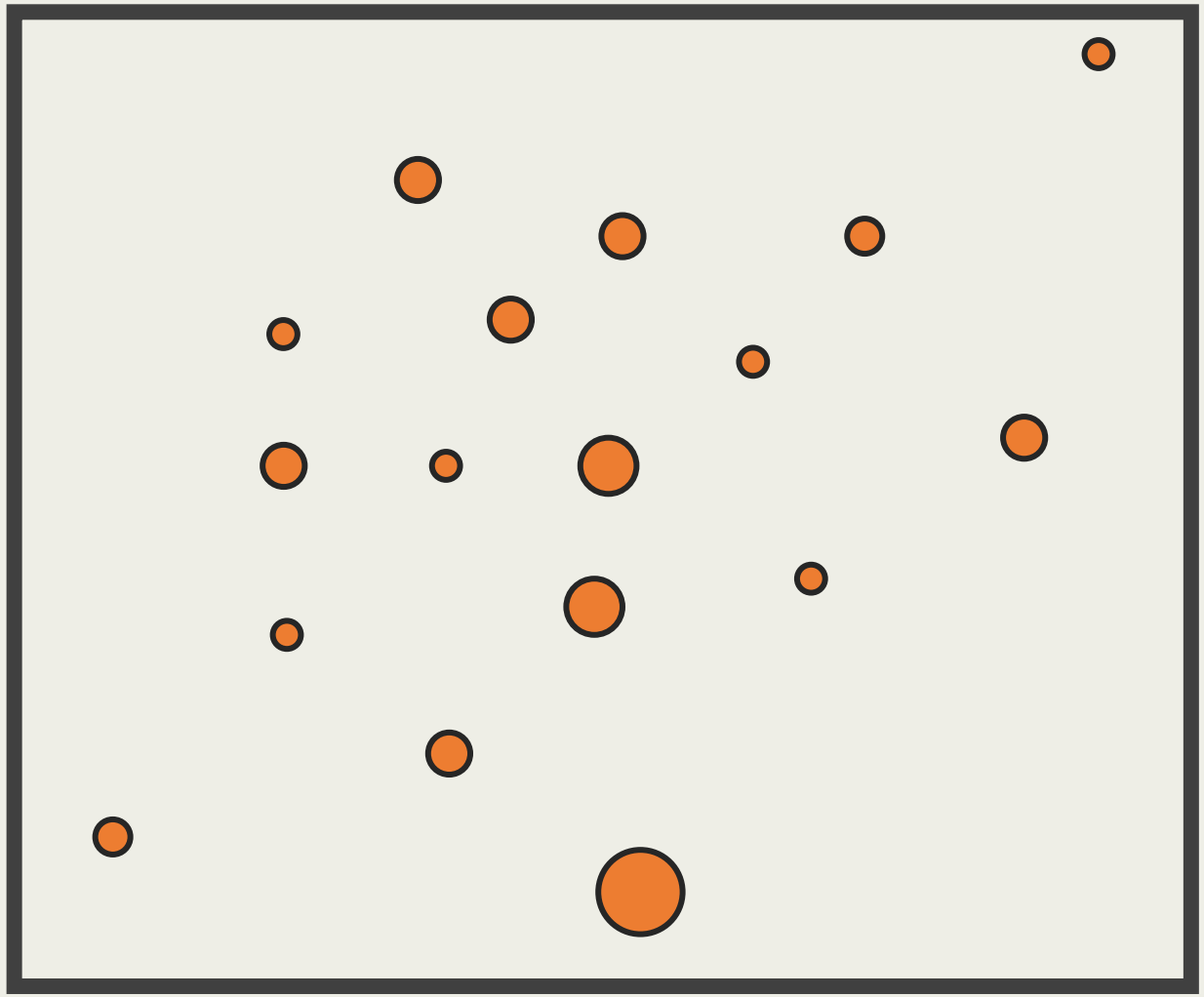

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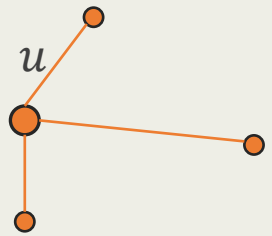

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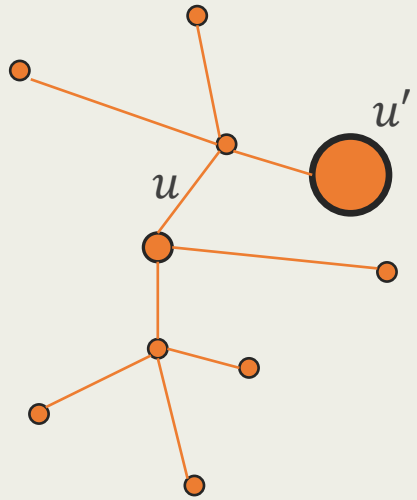

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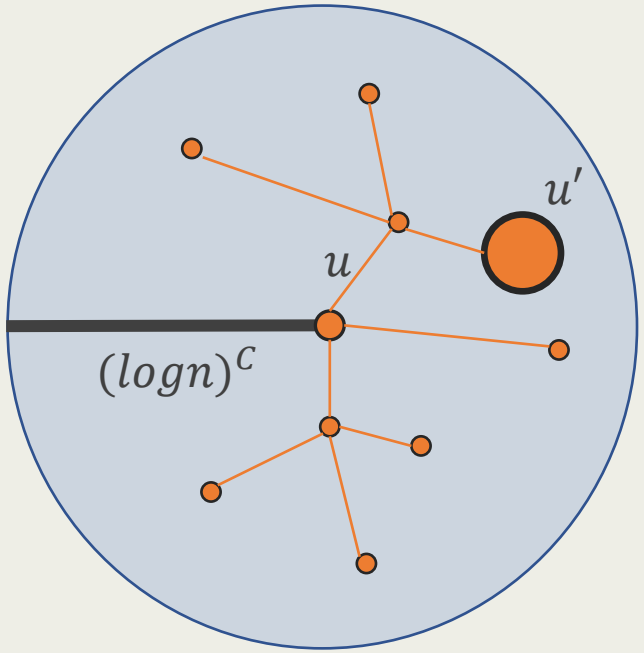

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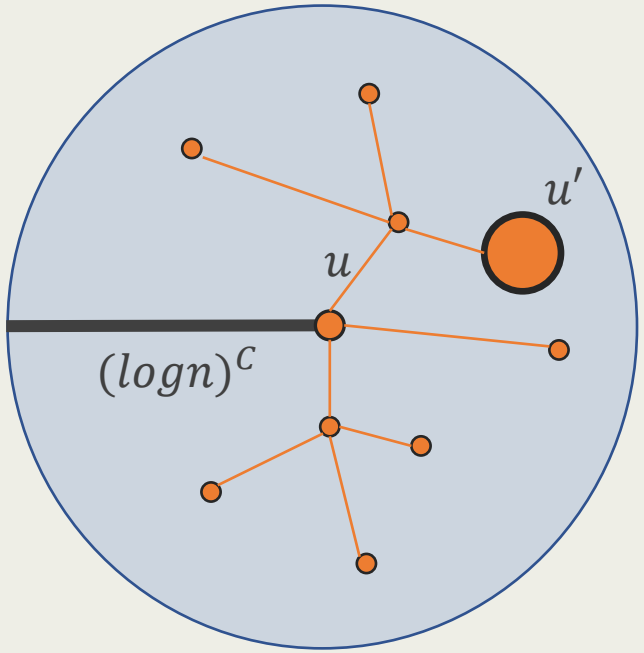

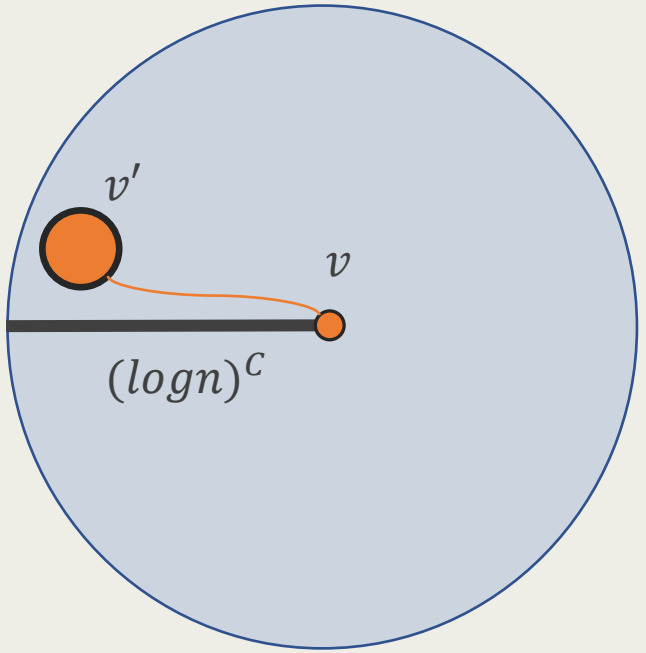

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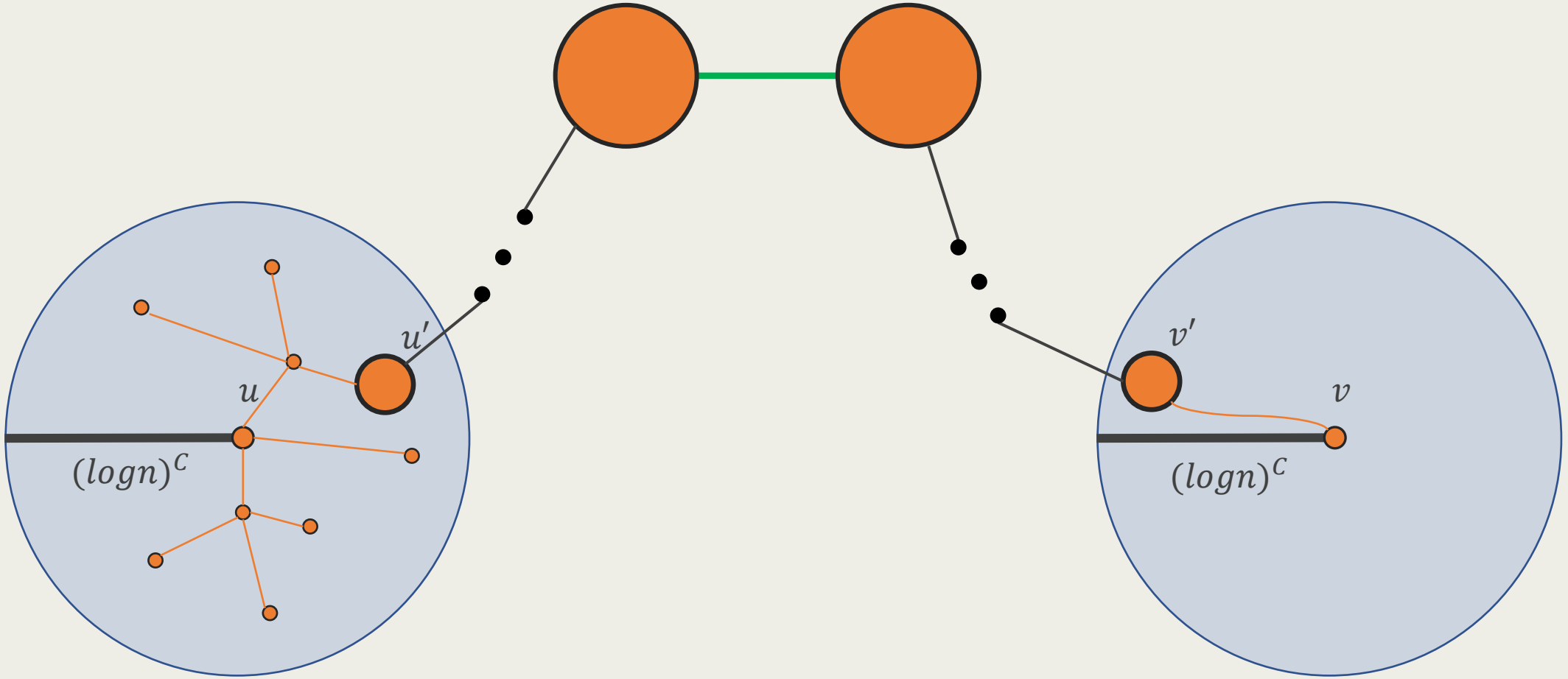

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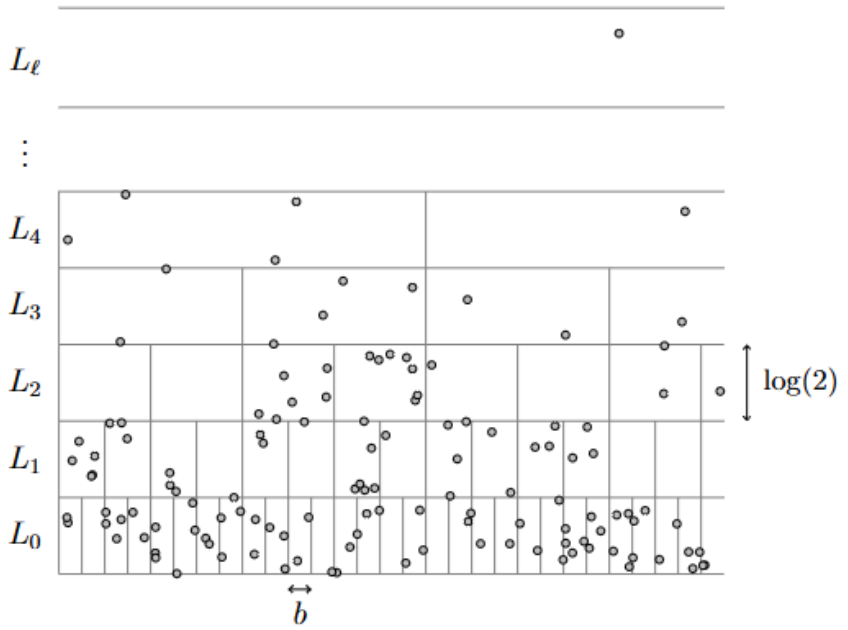

Figure 3: Partitioning $\mathcal{E}_R$ with boxes. All layers except L_ℓ have height log(2). The boxes in layer L_i have width $2^i b$, where $b \in [\frac{1}{6}, \frac{1}{3})$ is the width of a box in L_0. The small circles serve as an example of V.

Figures from paper above

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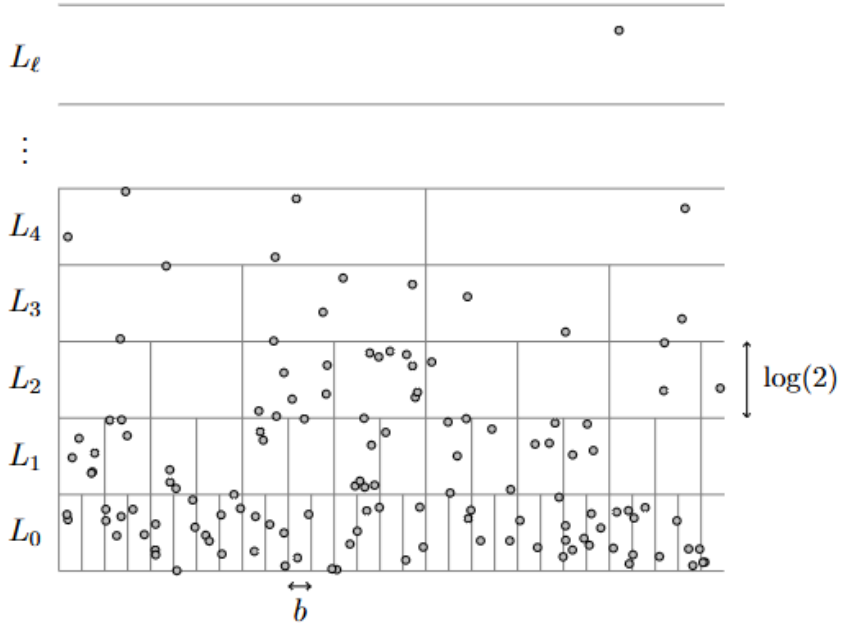

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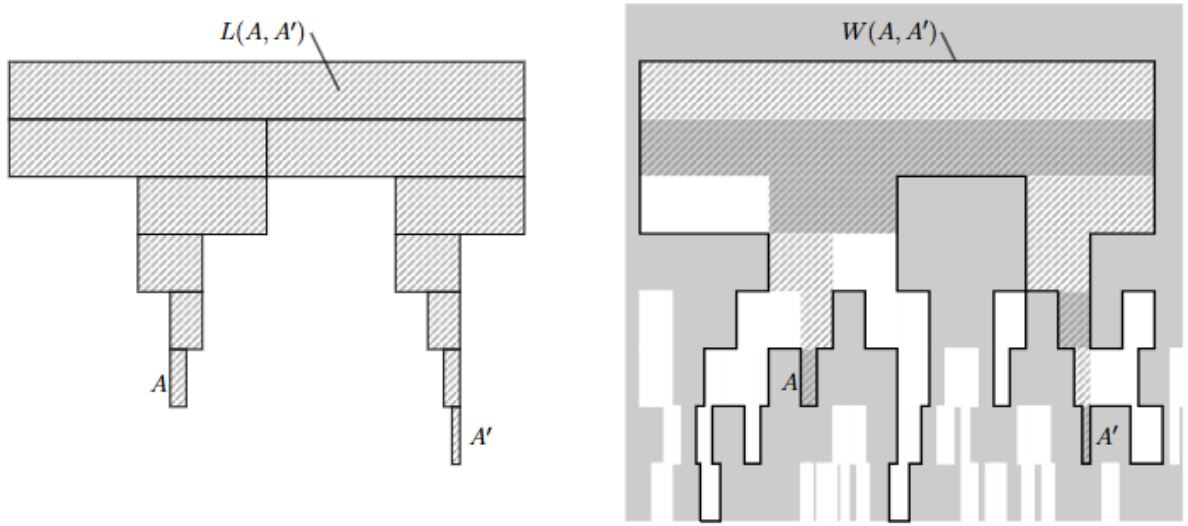

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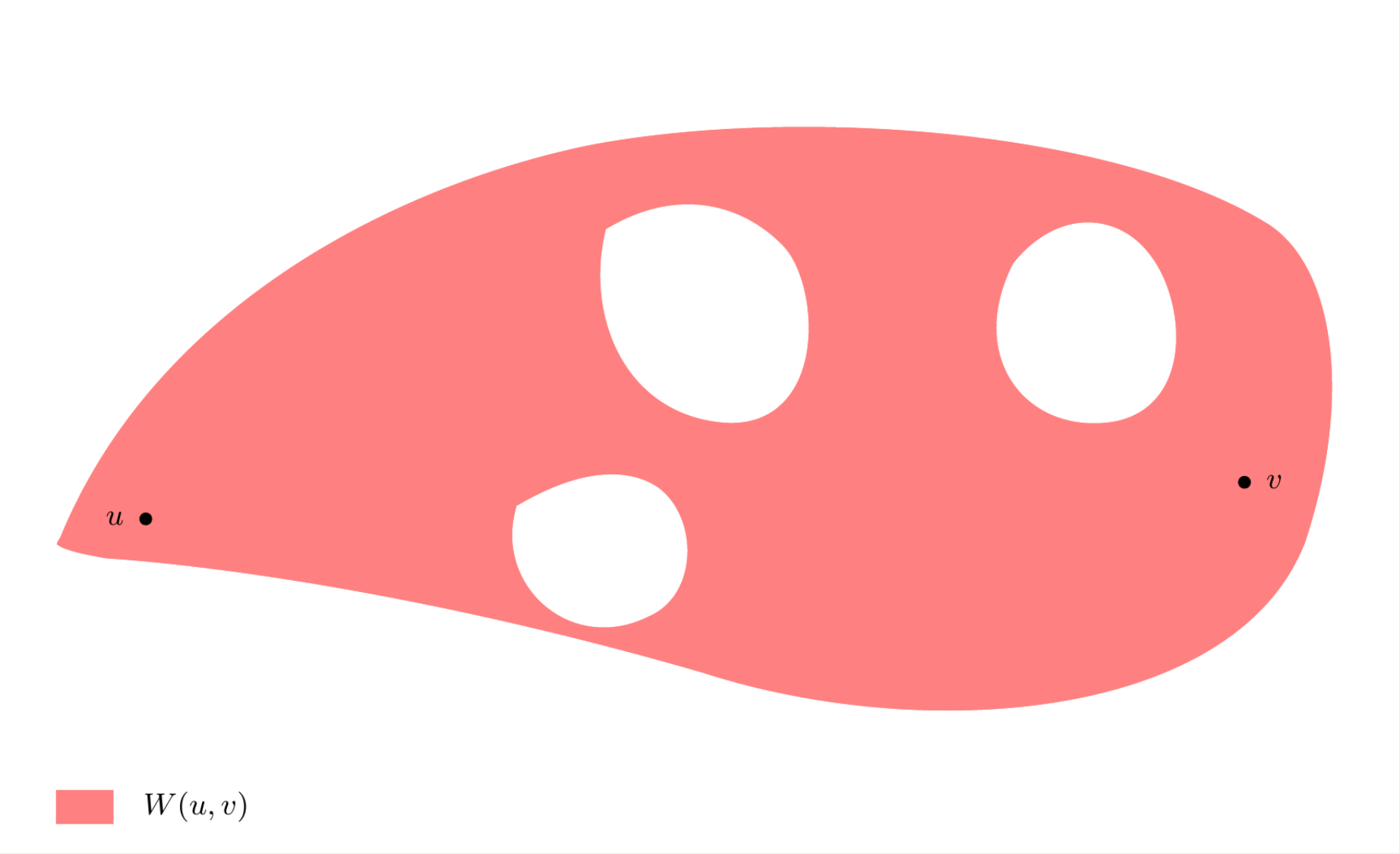

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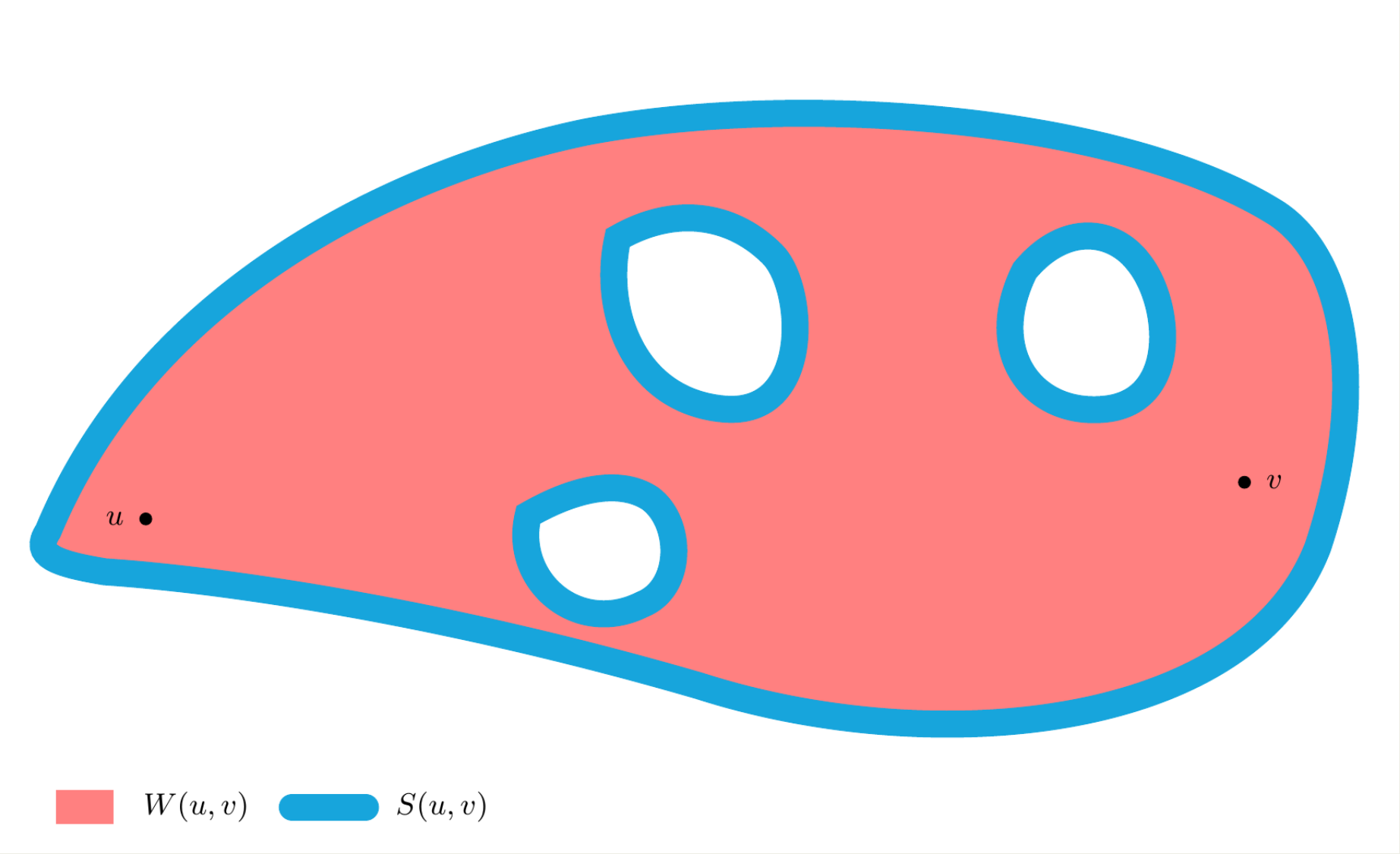

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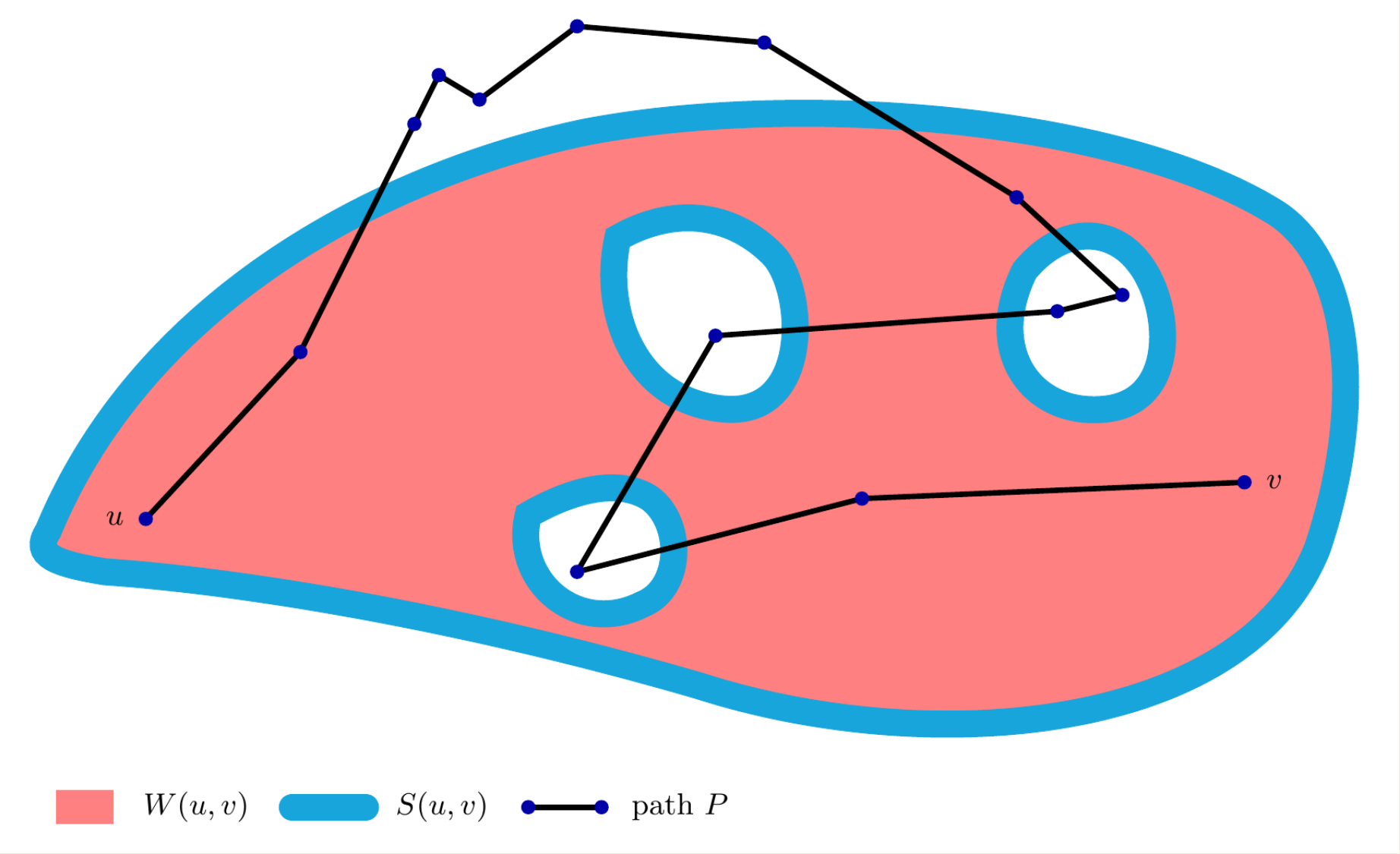

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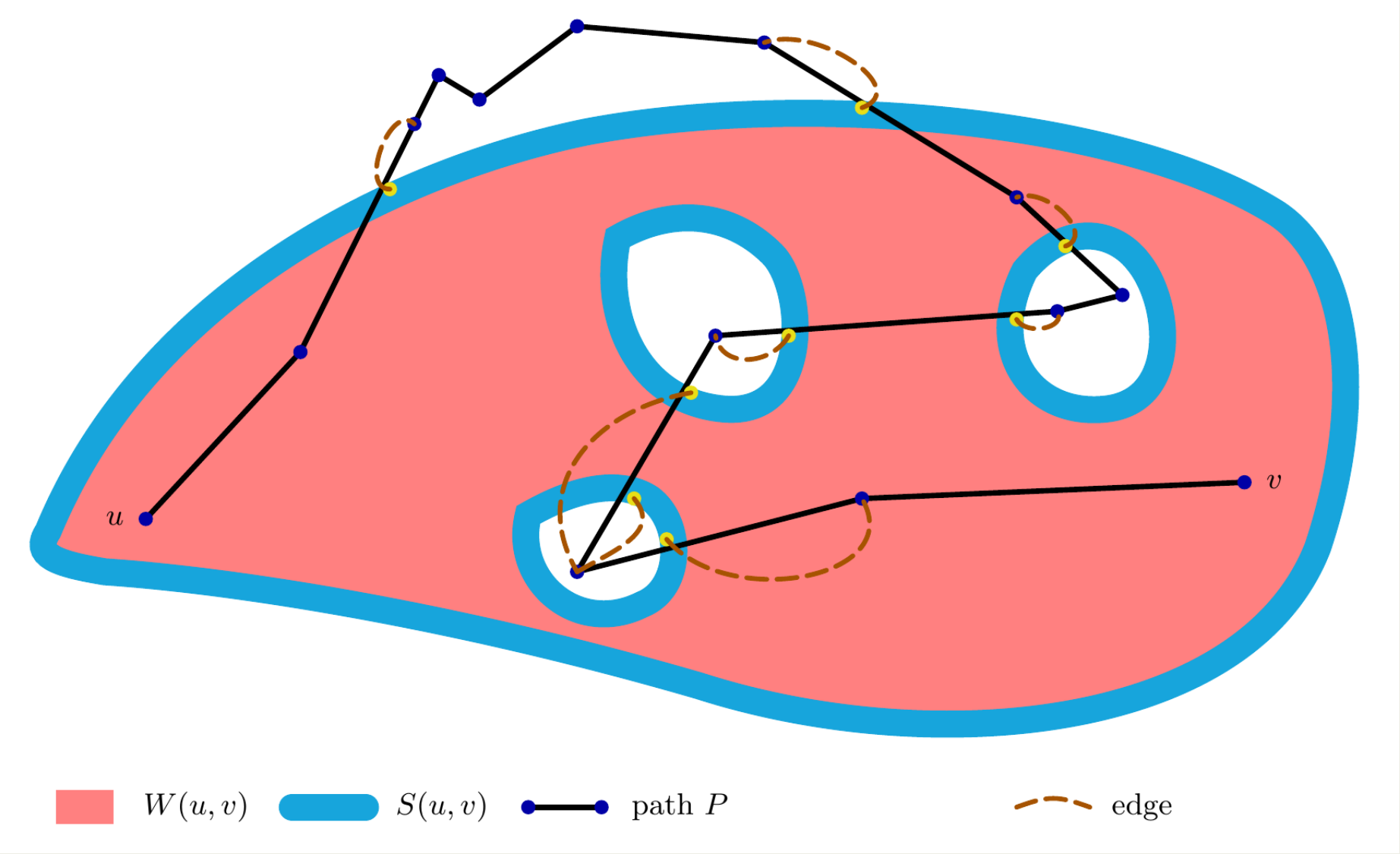

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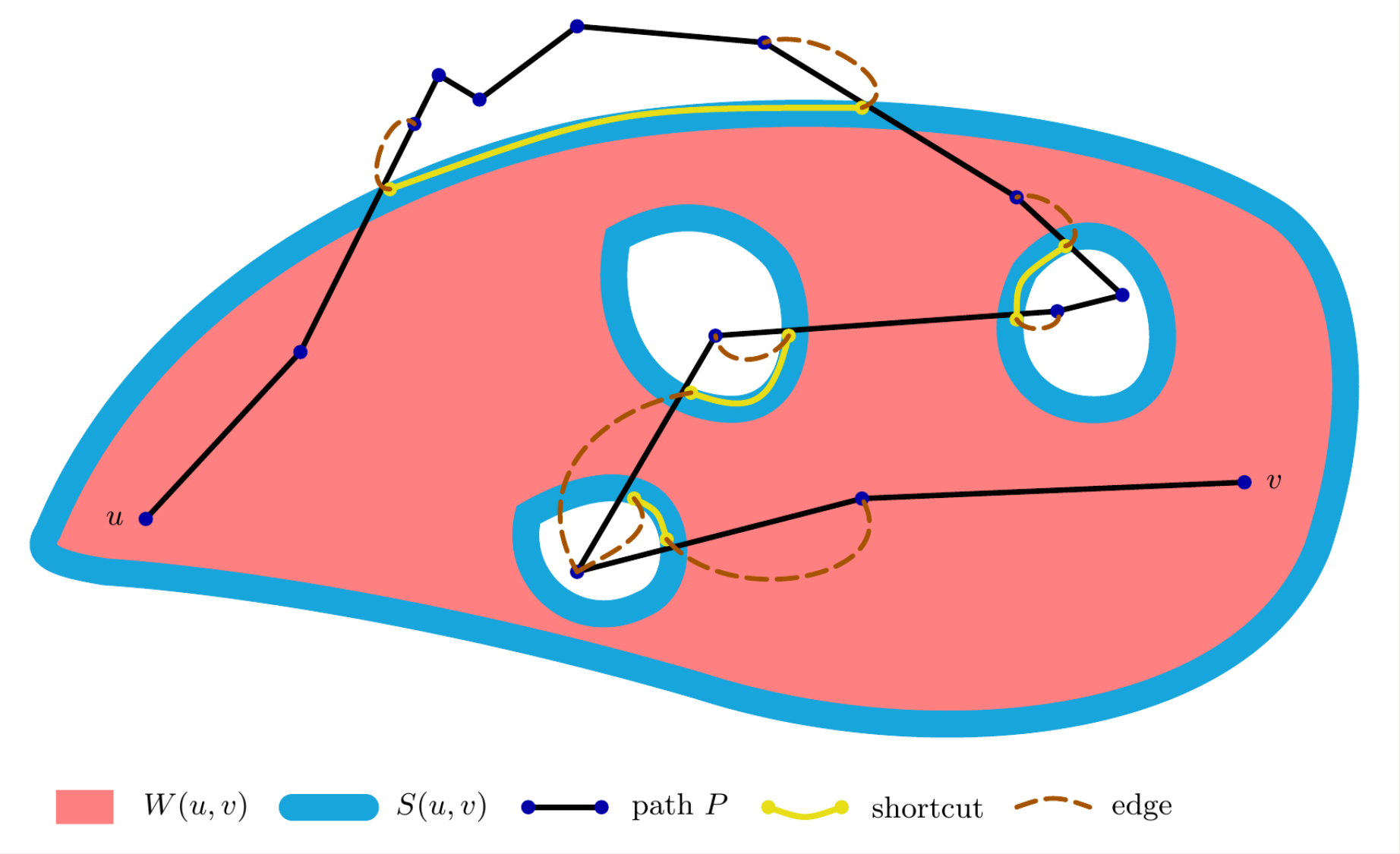

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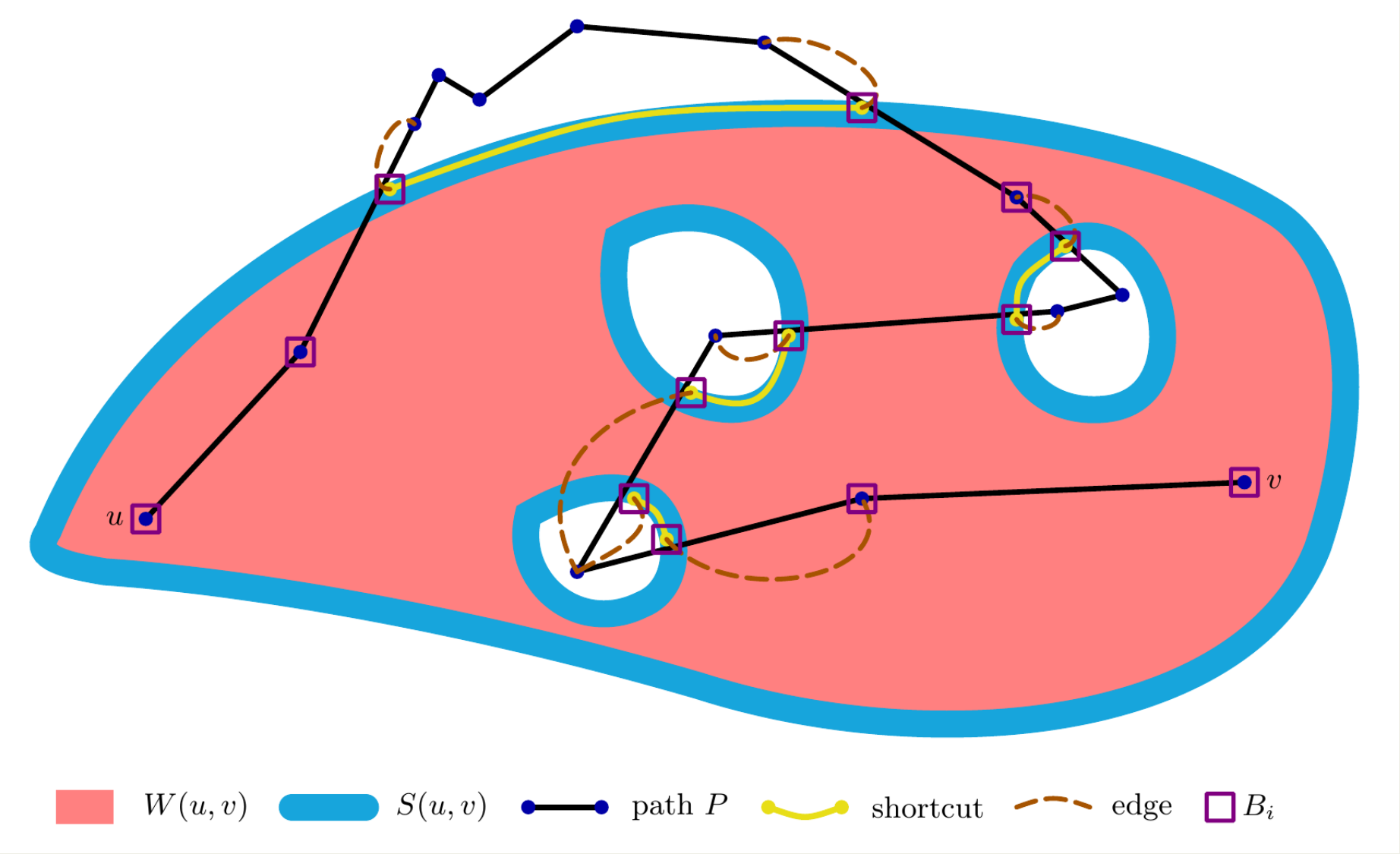

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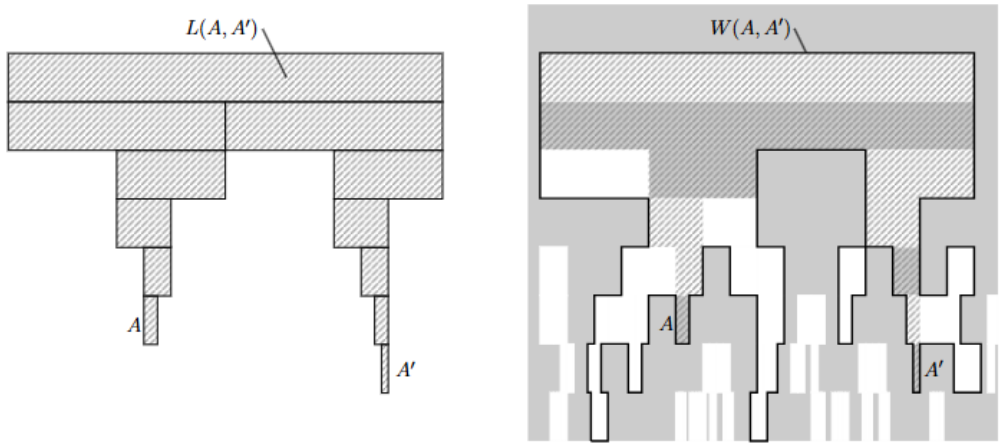

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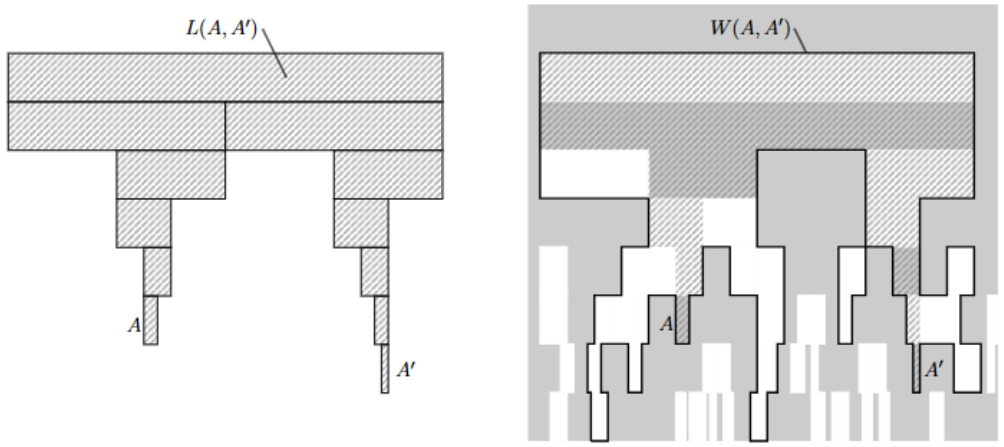

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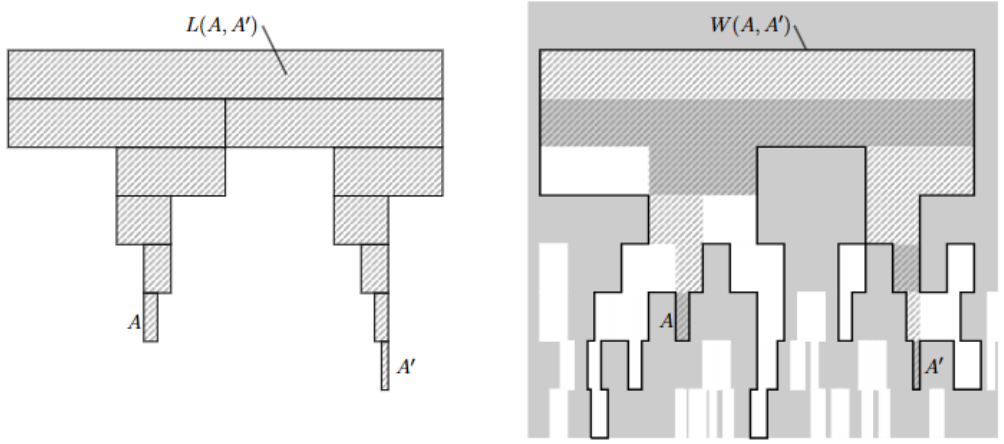

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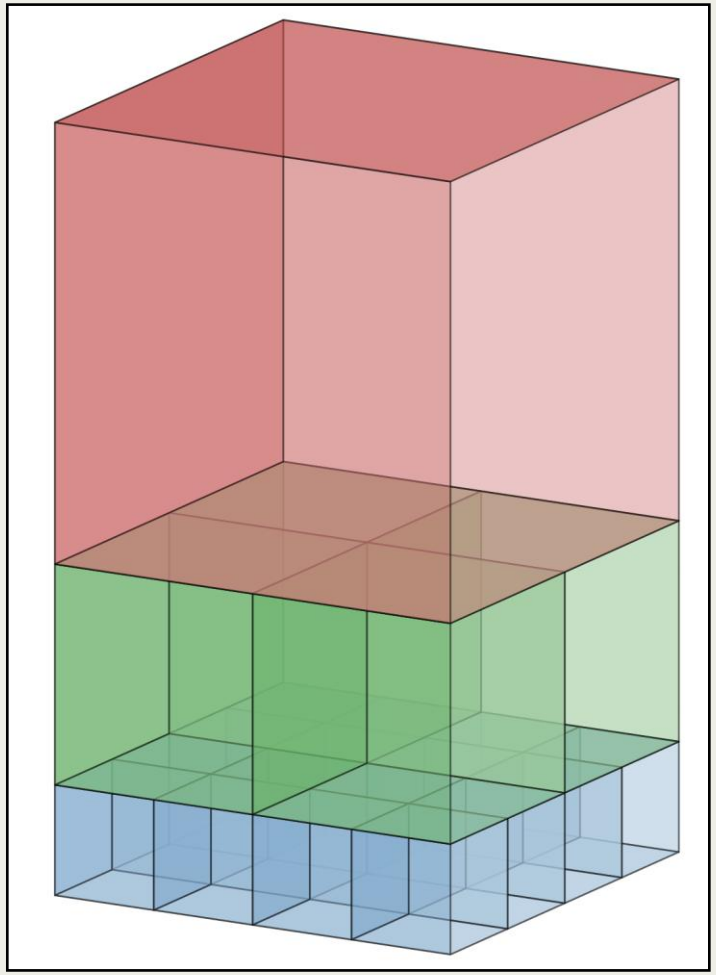
Proper solution: renormalize and consider "towers" of boxes. Call them active if they satisfy some nice properties that enable us to still "walk around" the resulting $W(u,v)$.



A random graph puzzle

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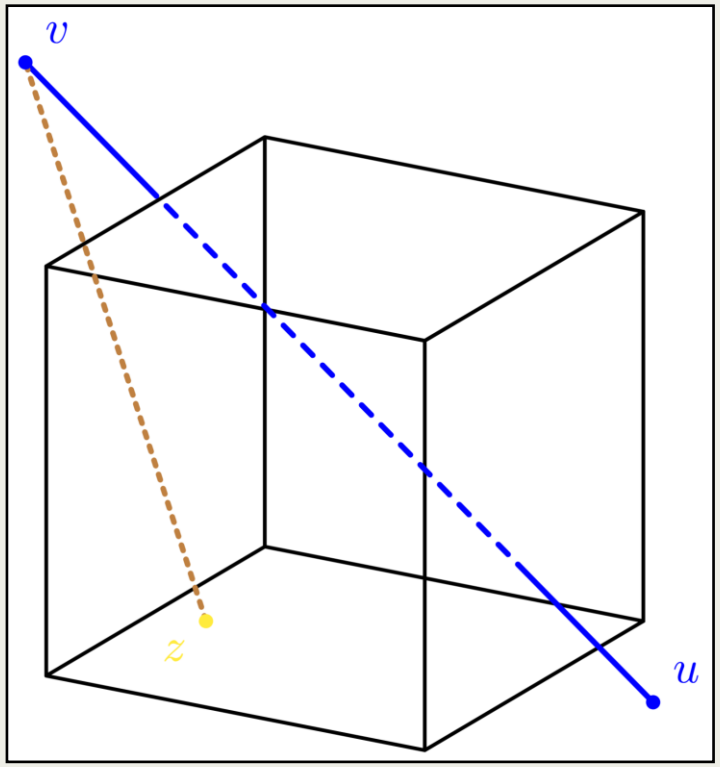

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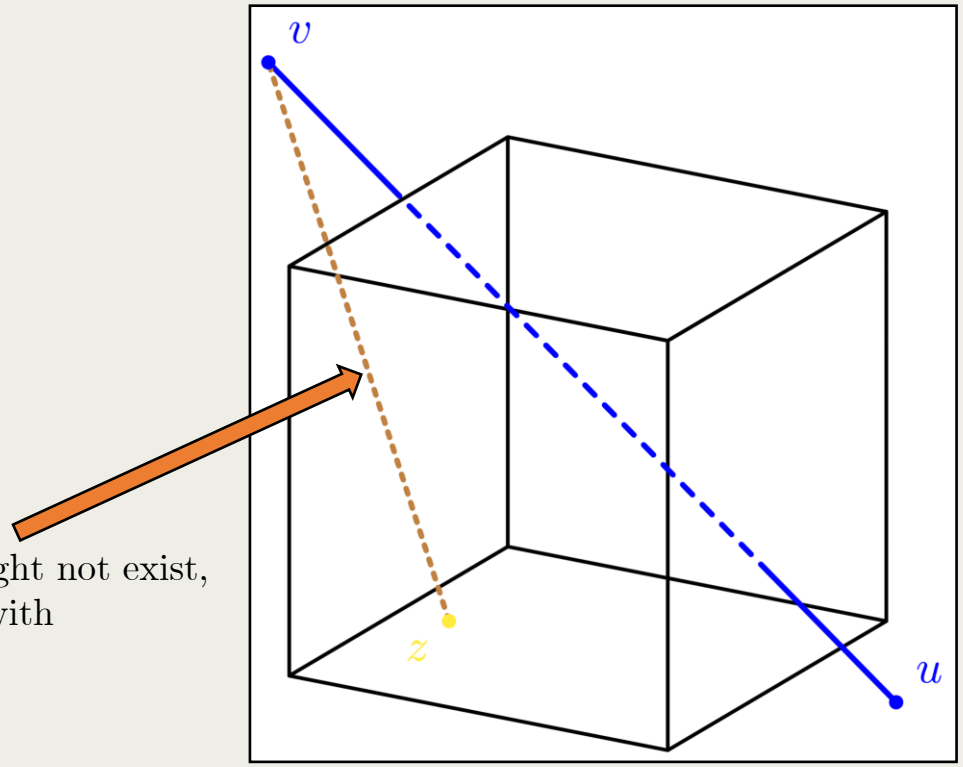

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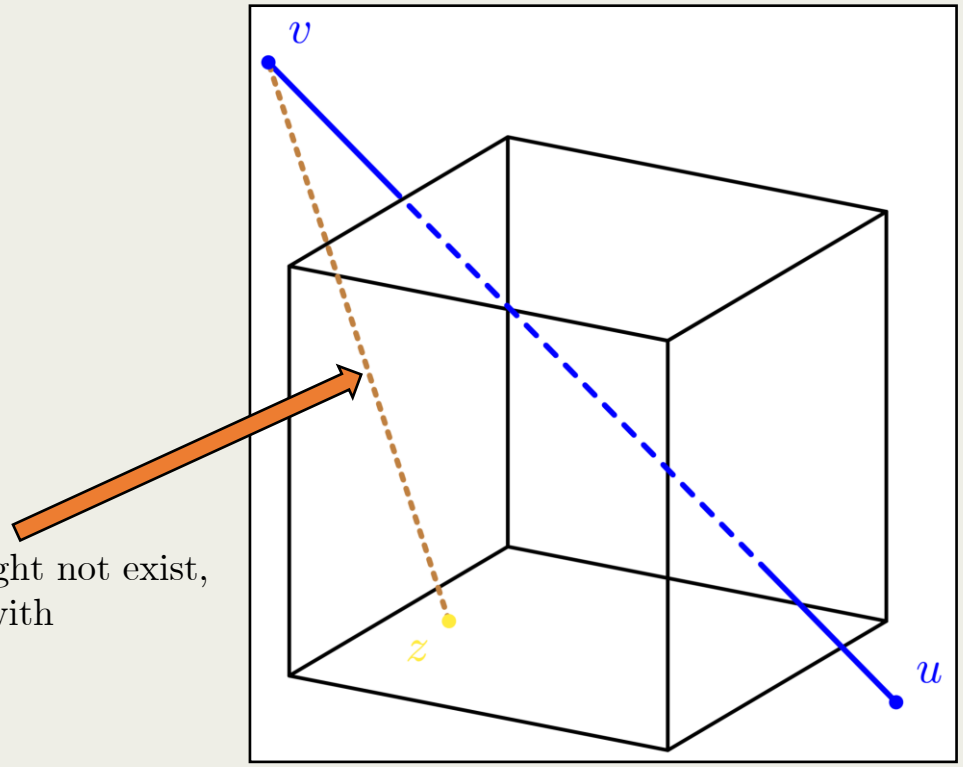

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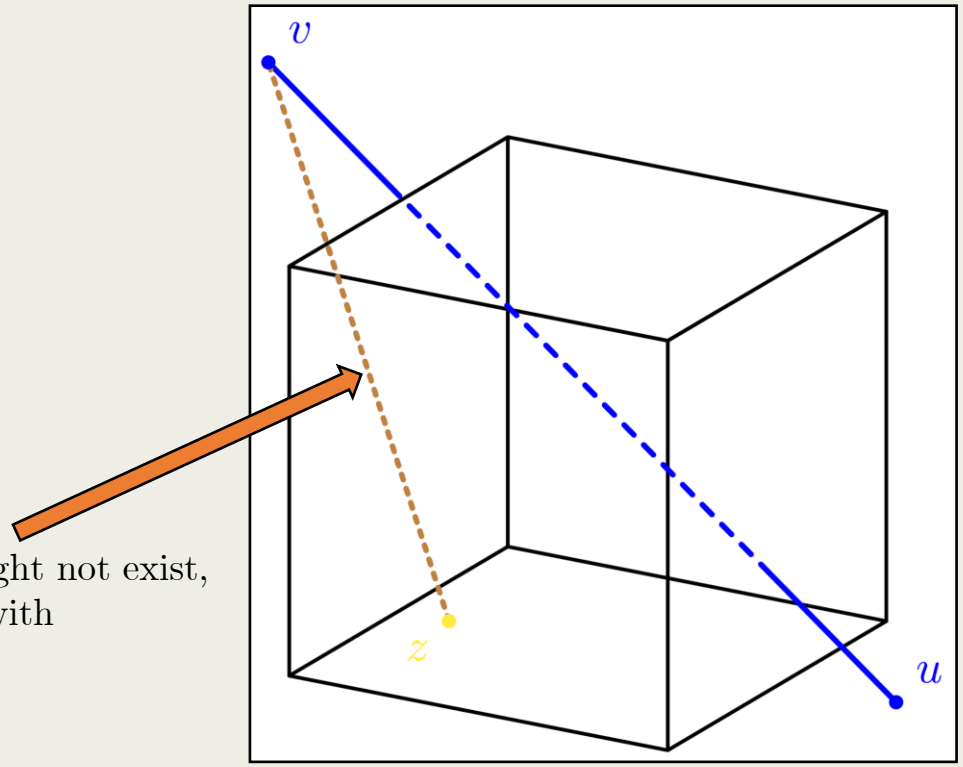

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Problem: we lose independence (endpoints could be anywhere).

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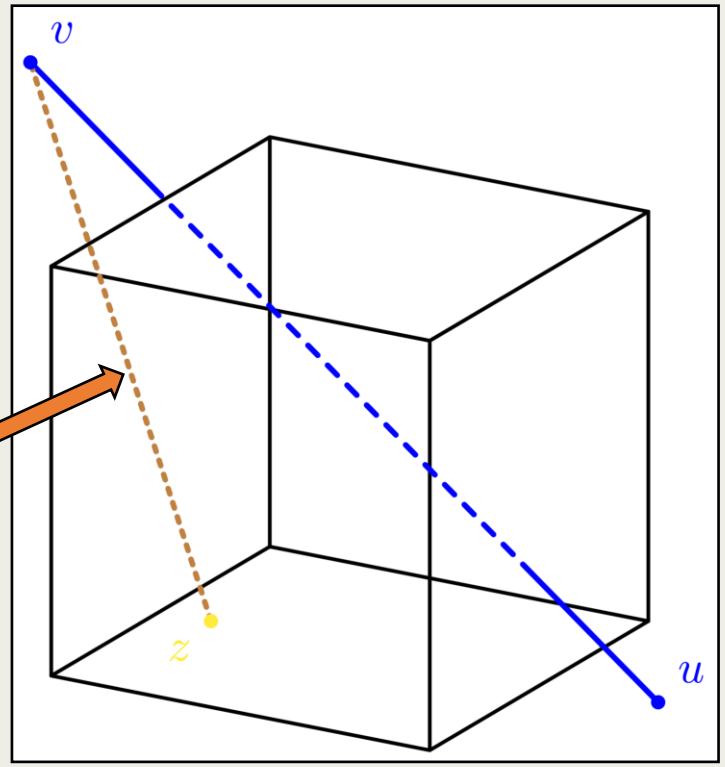

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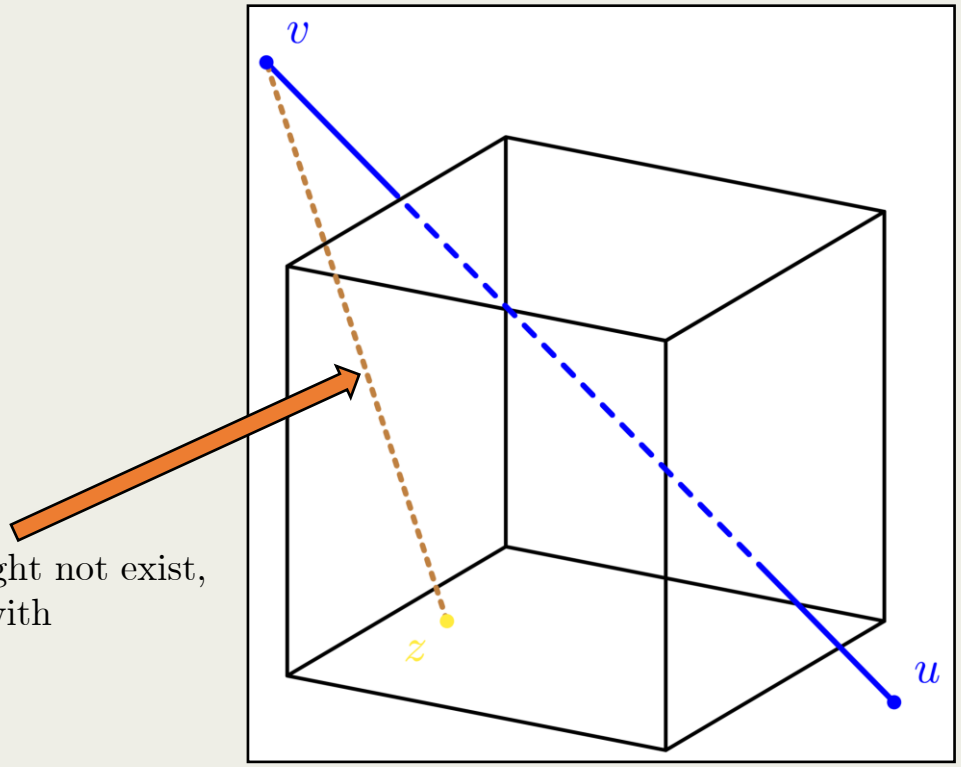

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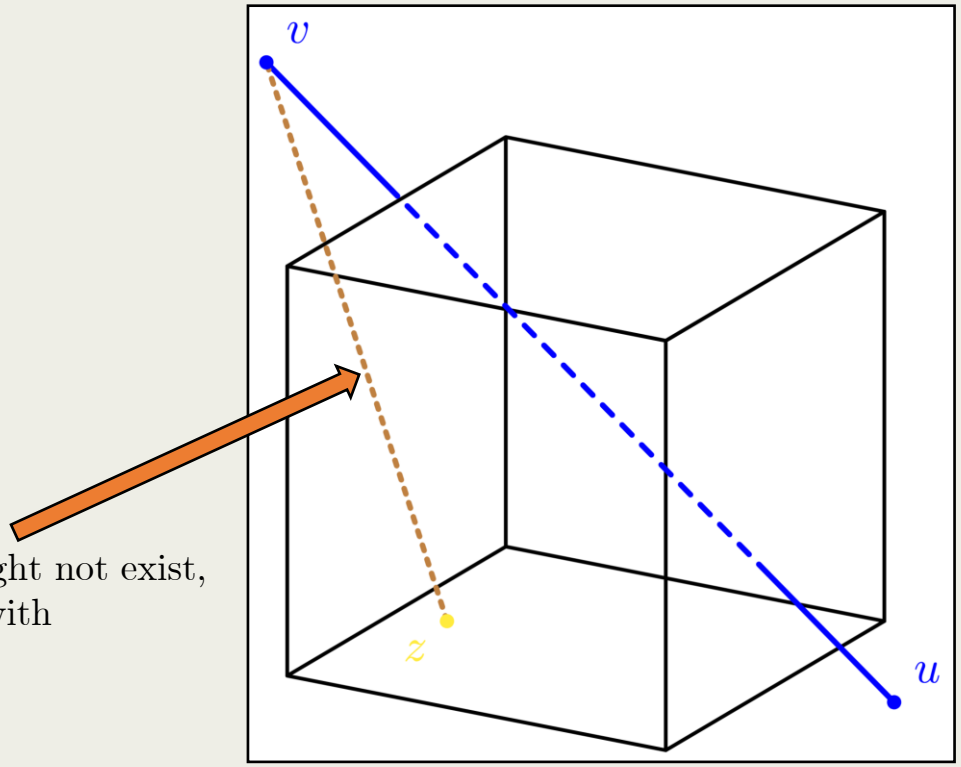

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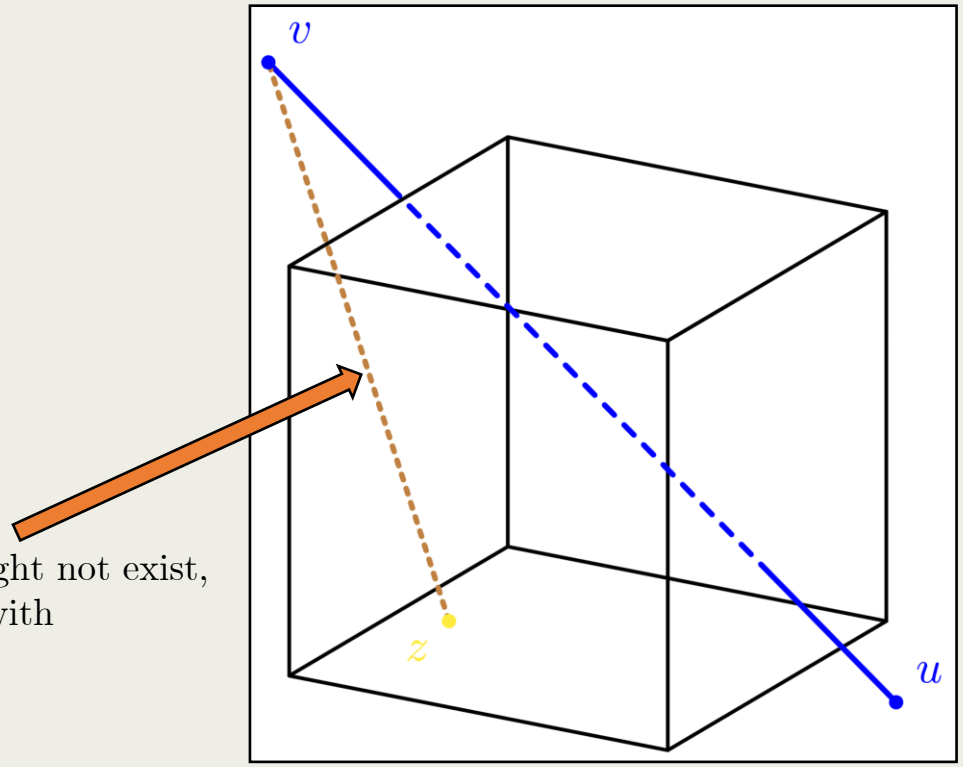

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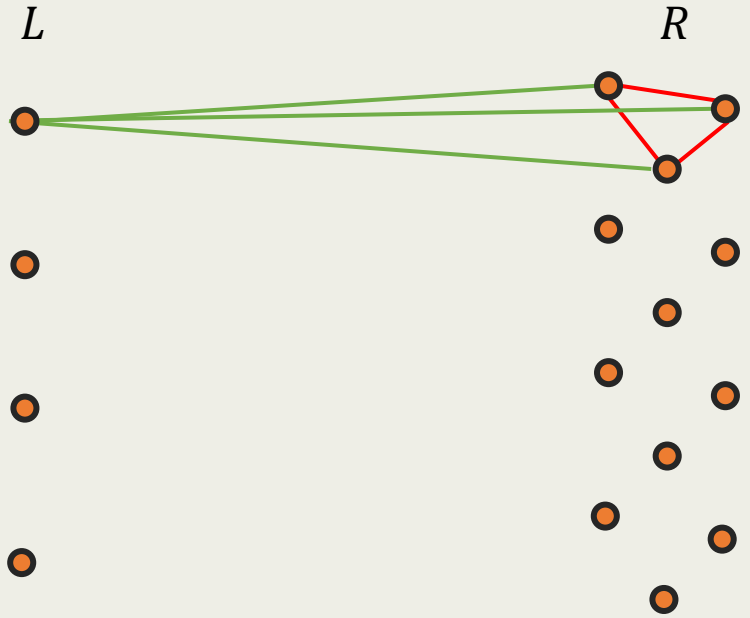

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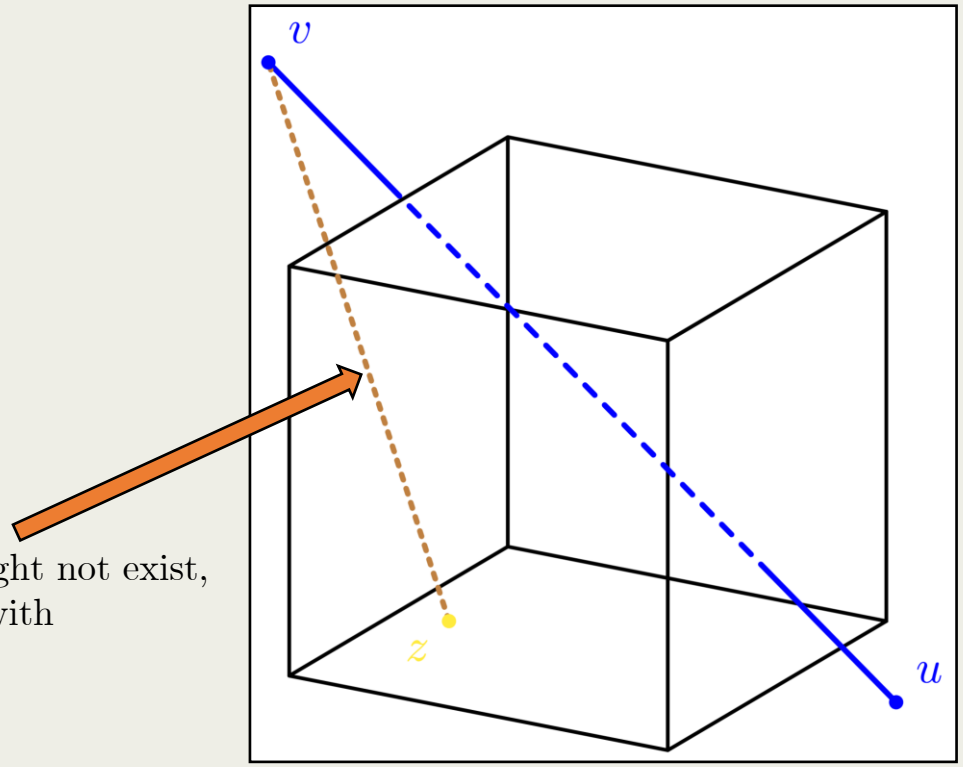

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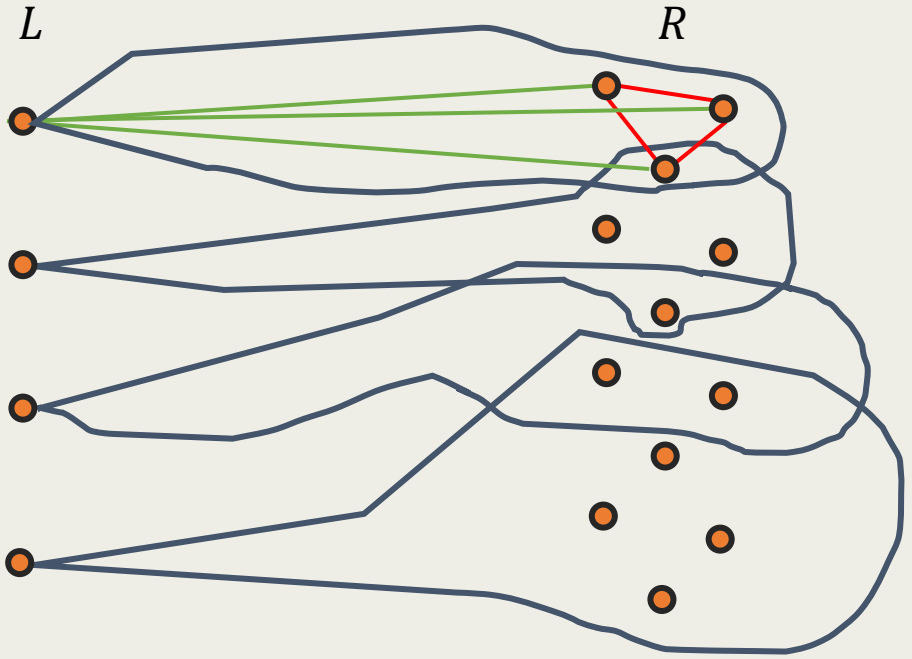

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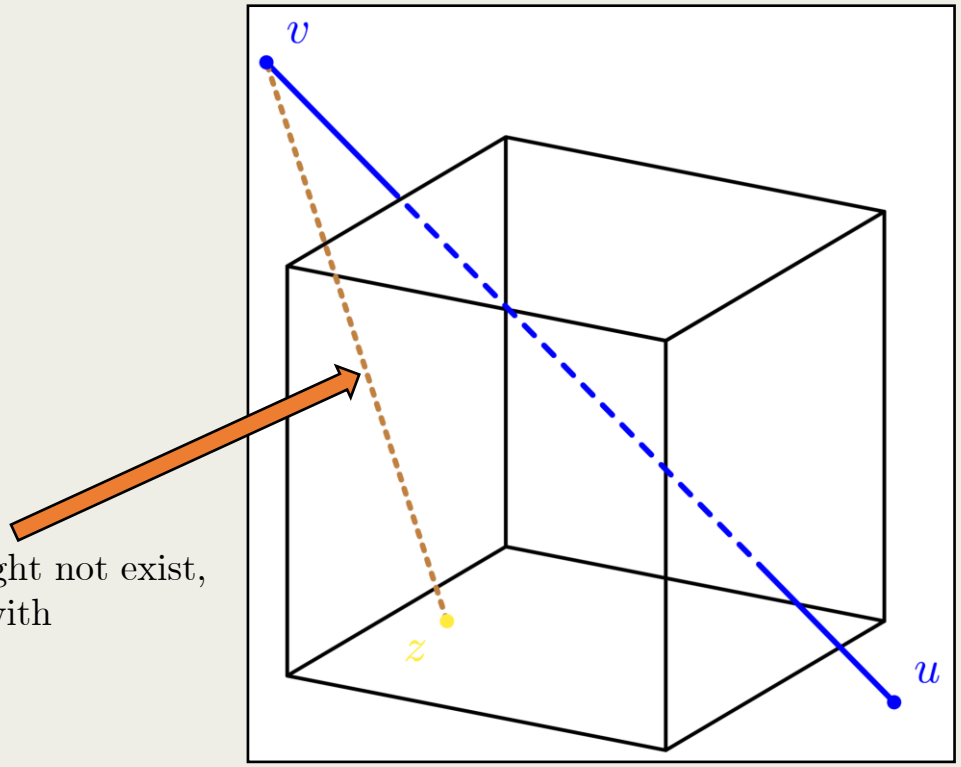

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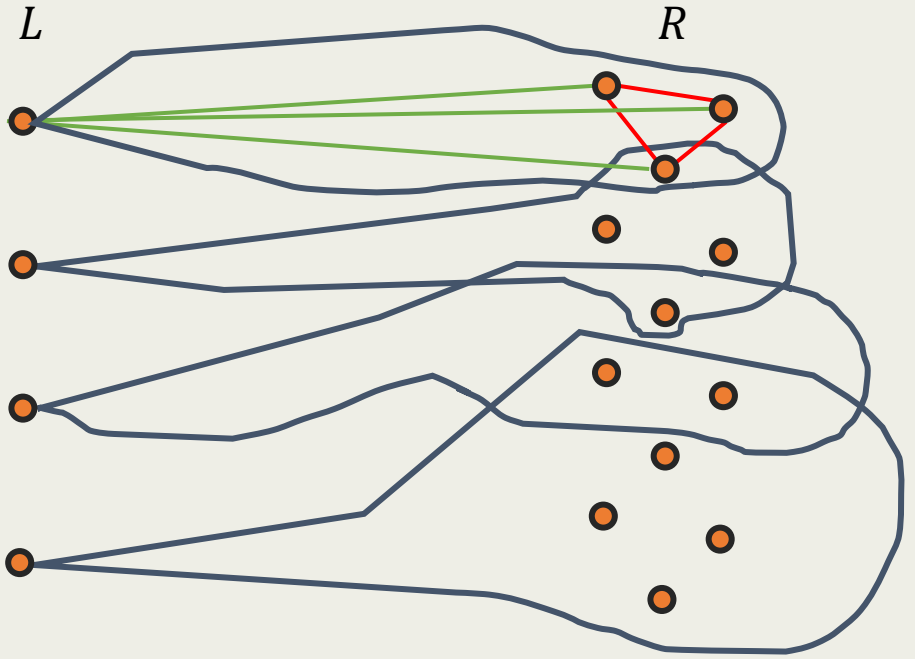

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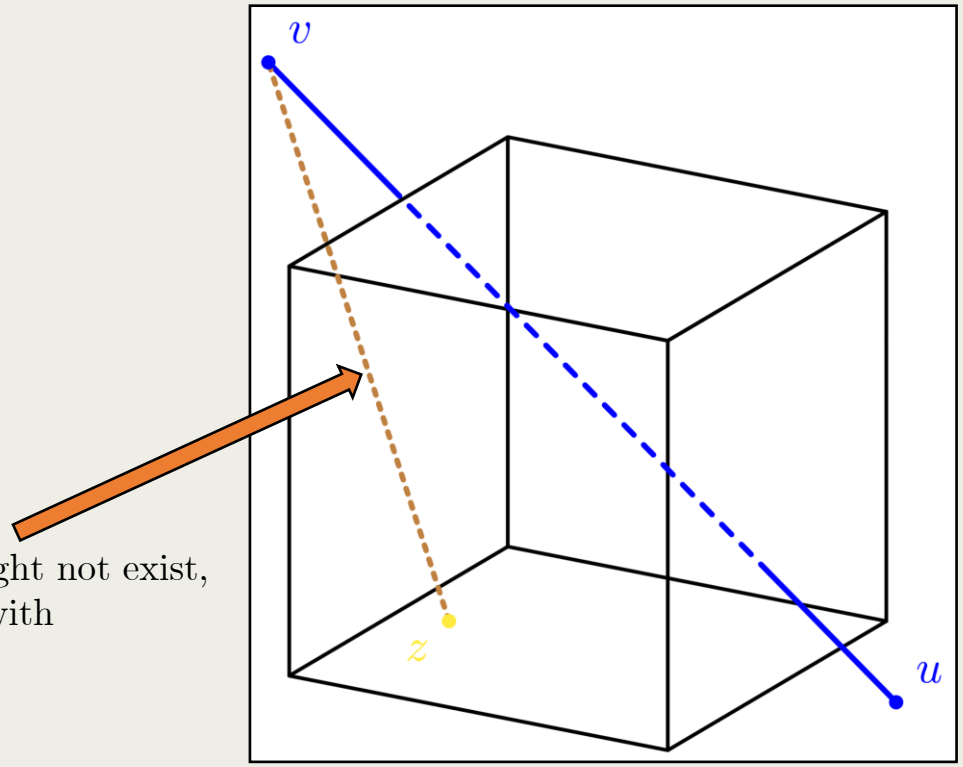

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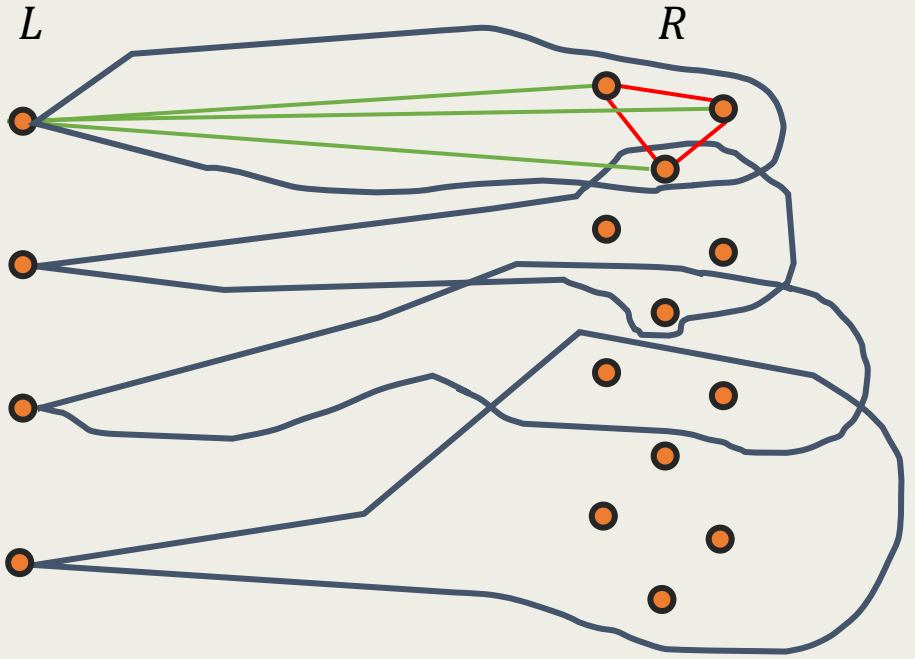

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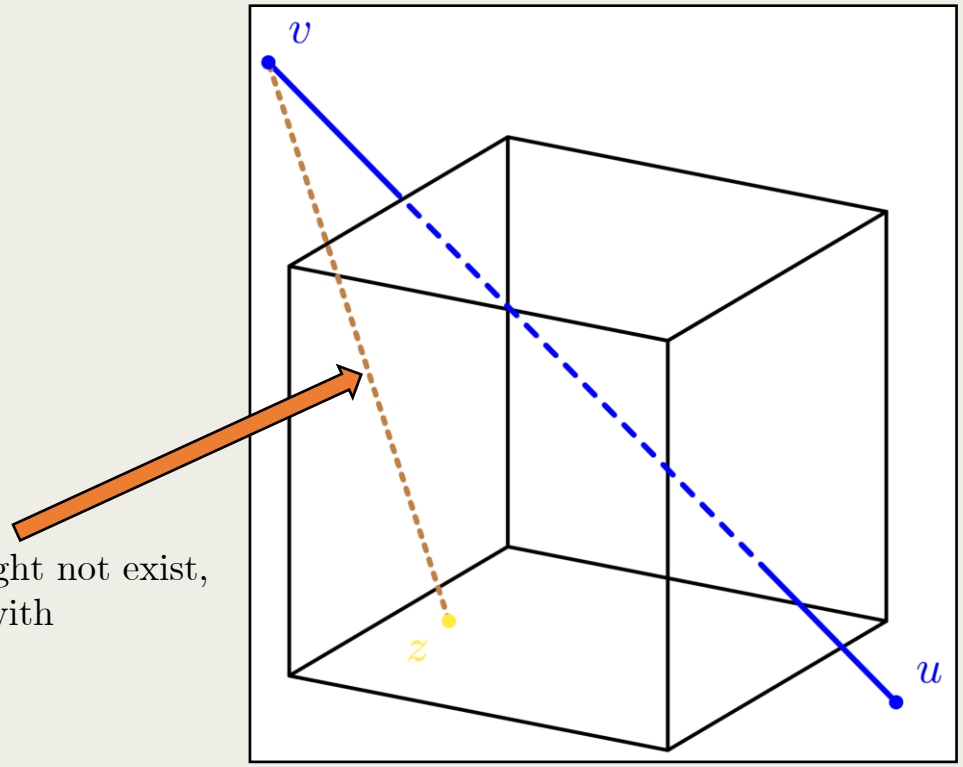

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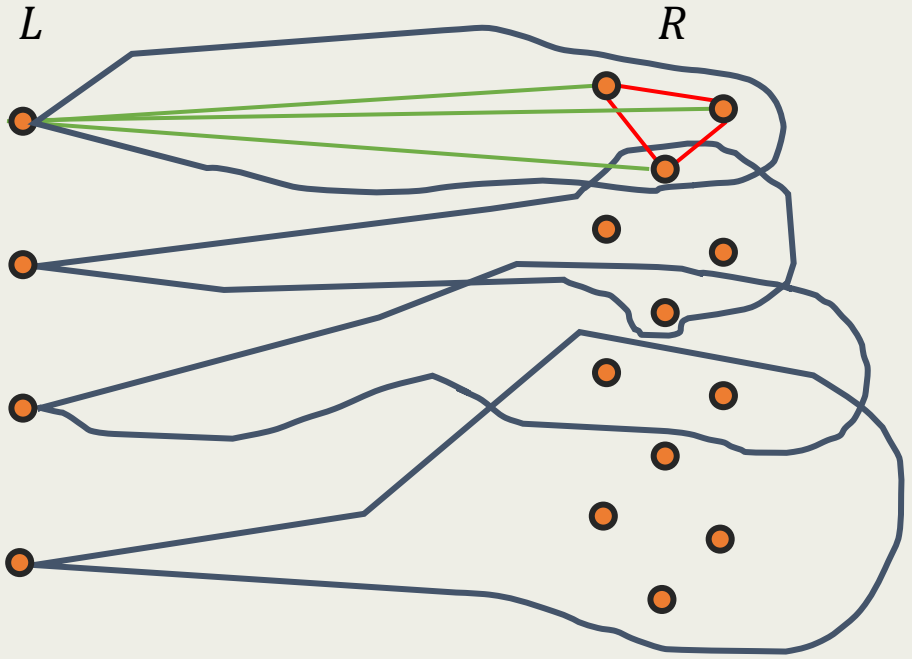

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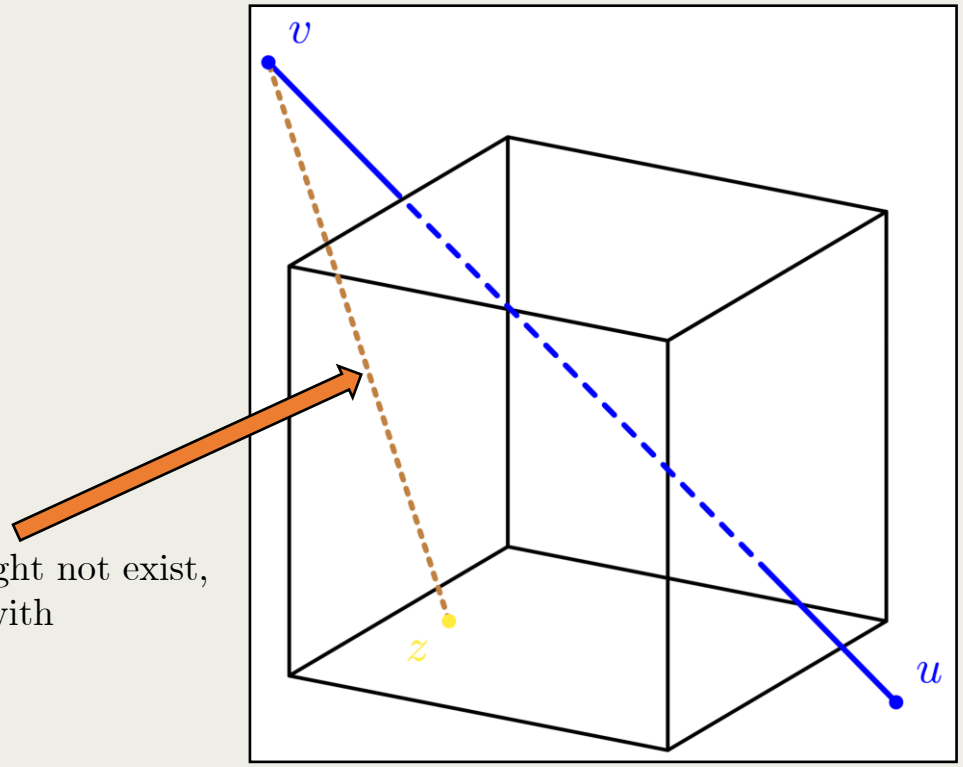

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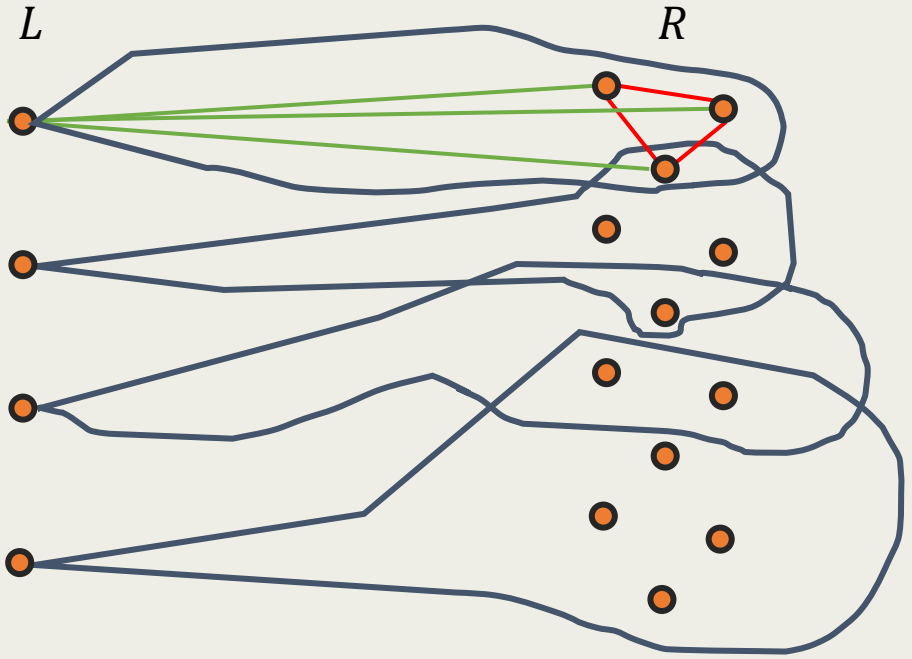

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