

Demystifying Codensity Monads

via Duality

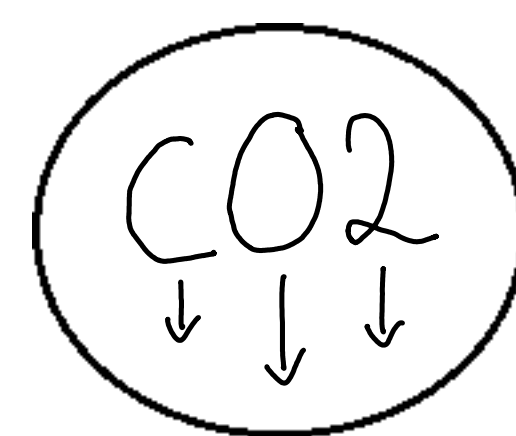
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STACS'26

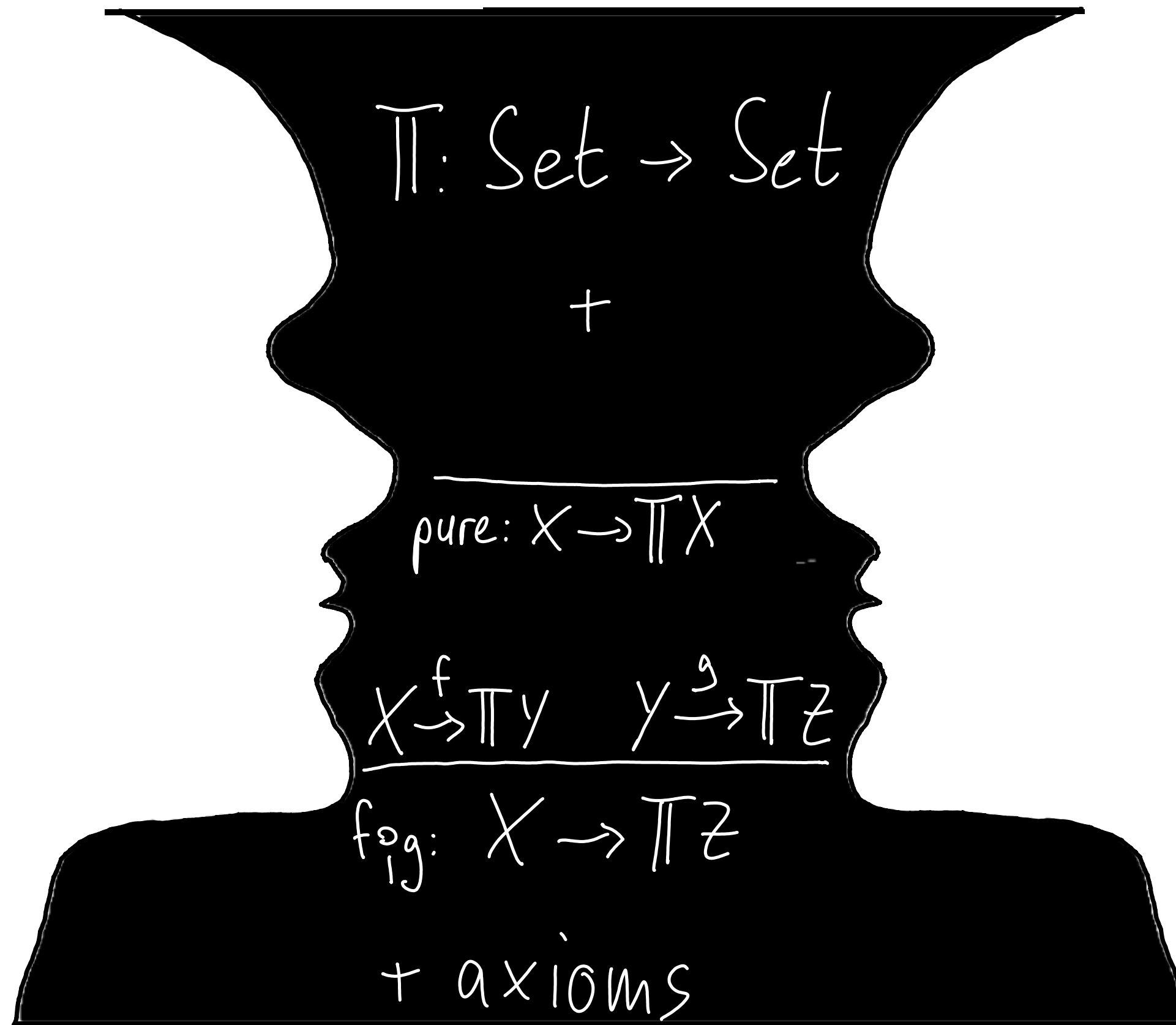
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low emission
presentation

Monads in Computer Science

Monad \mathbb{T} :



"Monads \cong Effects"

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Monads in Computer Science


Moggi, 1991, I&C

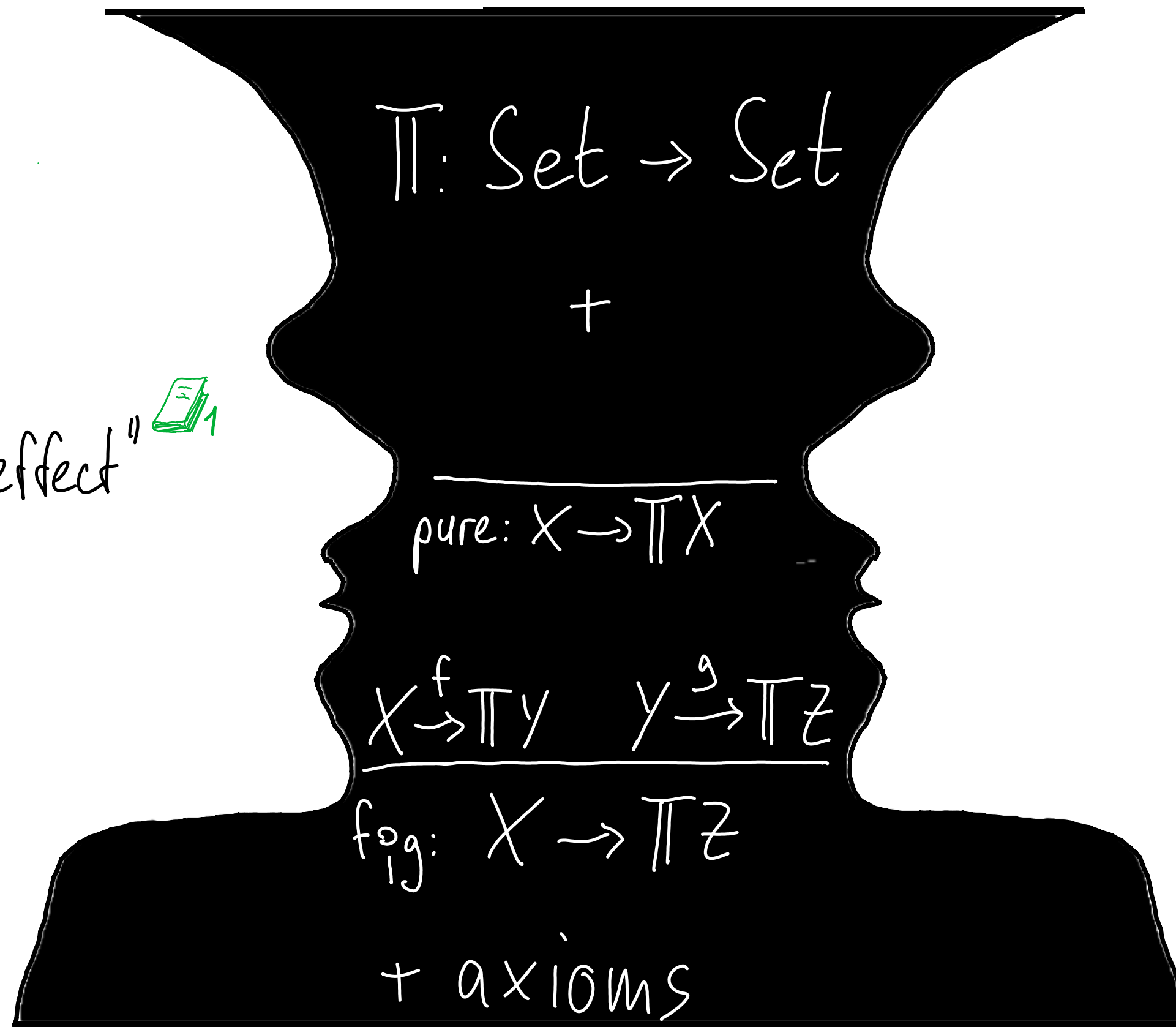
Effectful Computations

Monad \mathbb{T} :

Kleisli Maps

$$X \rightarrow \mathbb{T}Y$$

"map $X \rightarrow Y$ with \mathbb{T} -effect" 



"Monads \cong Effects"

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Effectful Computations

Monad \mathbb{T} :

Effectful Structures

Kleisli Maps

$$X \rightarrow \mathbb{T}Y$$

"map $X \rightarrow Y$ with \mathbb{T} -effect" 

$$\mathbb{T}: \text{Set} \rightarrow \text{Set}$$

+

Algebras

$$\mathbb{T}A \xrightarrow{a} A \quad (\text{axioms})$$

"evaluate \mathbb{T} -effects"

$$\text{pure}: X \rightarrow \mathbb{T}X$$

$$X \xrightarrow{f} \mathbb{T}Y \quad Y \xrightarrow{g} \mathbb{T}Z$$

$$\text{f} \circ \text{g}: X \rightarrow \mathbb{T}Z$$

+ axioms

Syntax

"Monads \cong Effects"

Semantics

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Monads: Examples

ΠX

$$P_f X = \{A \subseteq X \mid A \text{ finite}\}$$

$$X^* = \{x_1 \cdots x_n \mid x_i \in X\}$$

$$D X = \left\{ \sum_{i=1}^n r_i x_i \mid r_i \in [0, 1], x_i \in X, \sum_{i=1}^n r_i = 1 \right\}$$

Computations

nondeterminism

list of results

probabilities

Algebras

semilattices

monoids

convex sets

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