The Complexity of Resilience for Digraph Queries

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joint work with Manuel Bodirsky

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STACS
10 March 2026

ERC Synergy Grant POCOCOP (GA 101071674)

FWF Grant COVID (10.55776/ESP6949724)

Resilience of queries

database – a relational structure $\mathfrak{A}$

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Input: a finite database $\mathfrak{A}$, $u \in \mathbb{N}$
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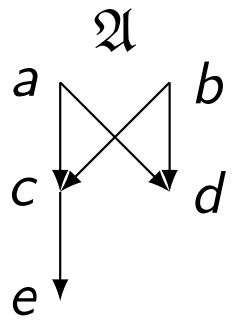

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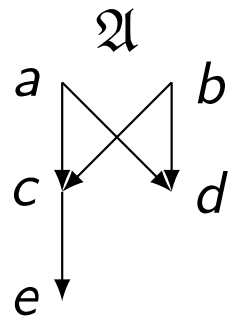

Goal: Classify complexity of resilience for all q (always in NP).

Translation to a dual problem

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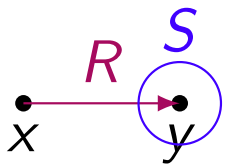
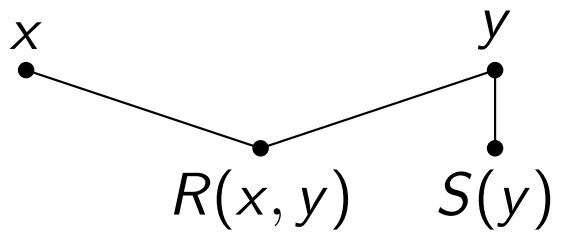


canonical structure



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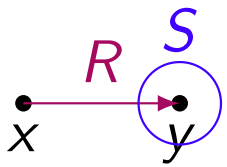

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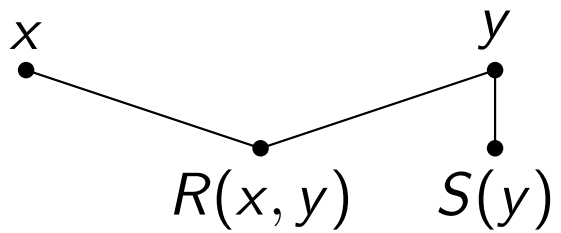

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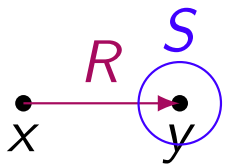

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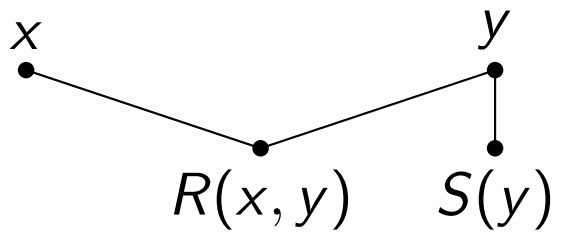

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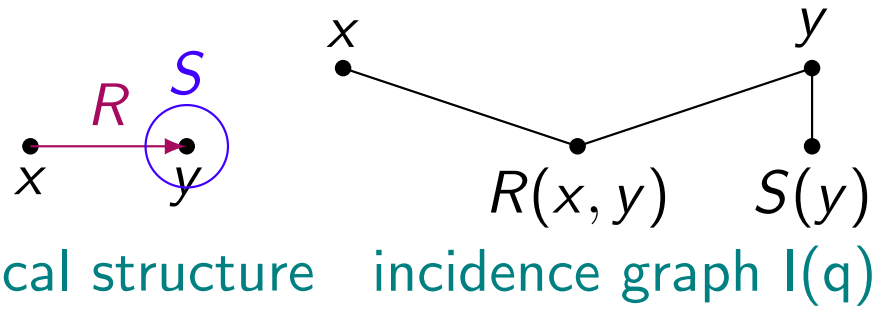

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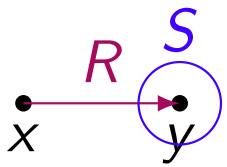

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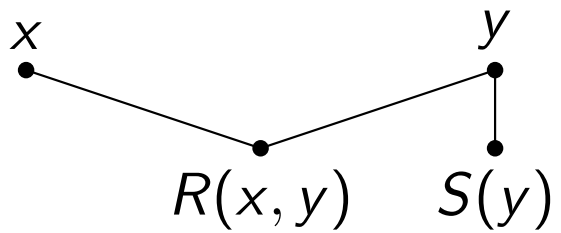

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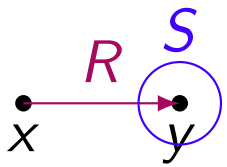

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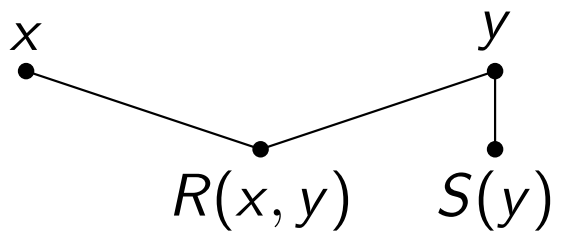

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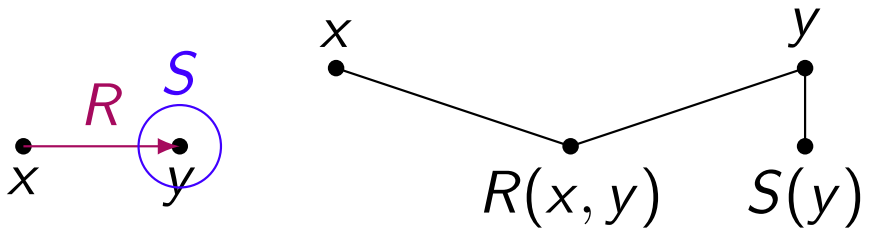

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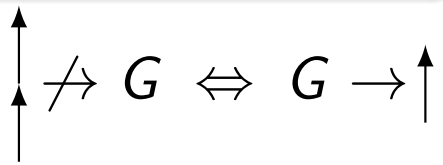
Example: For every finite directed graph G we have:



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Definition (VCSP(Δ))

Input: $u \in \mathbb{Q}$, an expression

$$\phi(x_1, \ldots, x_n) = \sum_i \psi_i,$$

where each ψ_i is an atomic τ-formula

Output: Is

$$\inf_{t \in D^n} \phi(t) \leq u \text{ in } \Delta?$$

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query q with $I(q)$ connected (WLOG) $\rightsquigarrow$ obtain the dual structure $\mathfrak{C}_q$ $\rightsquigarrow$ turn it into a valued structure Δ_q with cost functions taking values 0 and 1

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resilience problem for q_p = VCSP(Δ_{q_p})
= Max-Cut problem for directed multigraphs (NP-complete)

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Fact (Bodirsky, S., Lutz '24): If $\mathrm{Aut}(\Gamma)$ is oligomorphic and Γ pp-constructs Δ, then $\mathrm{VCSP}(\Delta)$ reduces to $\mathrm{VCSP}(\Gamma)$ in polynomial time.

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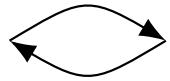

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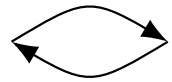

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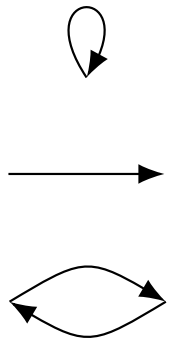

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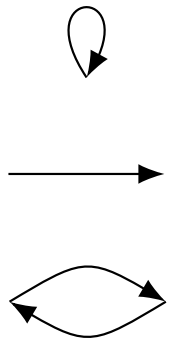

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Theorem (Bodirsky, S. '26)

Let q be a minimal digraph query. If $q \in \{q_\ell, q_e, q_c\}$, then the resilience problem for q is in P. Otherwise, Δ_q pp-constructs 1-in-3-SAT and the resilience problem for q is NP-complete.

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Let q be a self-join variation of a self-join-free query q'. If q is minimal, then Δ_q pp-constructs $\Delta_{q'}$.

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Let q be a self-join-free query over a binary signature. If q contains an oriented cycle of length ≥ 3, then Δ_q pp-constructs 1-in-3-SAT.

Theorem (Bodirsky, S. '26)

Let q be a self-join variation of a self-join-free query q'. If q is minimal, then Δ_q pp-constructs $\Delta_{q'}$.

Corollary: The first theorem is true without the assumption that q is self-join-free.

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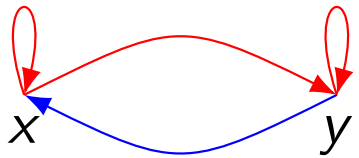
- Δ_{q_p} pp-constructs 1-in-3-SAT.
- We are done by the transitivity of pp-constructability.

Future of resilience

Next to-do: Prove a P vs. NP-complete complexity dichotomy for conjunctive queries over a binary signature.



Thank you for your attention

Funding statement: This research was funded in whole or in part by the Austrian Science Fund (FWF) 10.55776/ESP6949724 and by the European Union (ERC, POCOCOP, 101071674). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.